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Positive Definiteness of Paired Symmetric Tensors and Elasticity Tensors

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Abstract

In this paper, we consider higher order paired symmetric tensors and strongly paired symmetric tensors. Elasticity tensors and higher order elasticity tensors in solid mechanics are strongly paired symmetric tensors. A (strongly) paired symmetric tensor is said to be positive definite if the homogeneous polynomial defined by it is positive definite. Positive definiteness of elasticity and higher order elasticity tensors is strong ellipticity in solid mechanics, which plays an important role in nonlinear elasticity theory. We mainly investigate positive definiteness of fourth order three dimensional and sixth order three dimensional (strongly) paired symmetric tensors. We first show that the concerned (strongly) paired symmetric tensor is positive definite if and only if its smallest M-eigenvalue is positive. Second, we propose several necessary and sufficient conditions under which the concerned (strongly) paired symmetric tensor is positive definite. Third, we study the conditions under which the homogeneous polynomial defined by a fourth order three dimensional or sixth order three dimensional (strongly) paired symmetric tensor can be written as a sum of squares of polynomials, and further, propose several necessary and/or sufficient conditions to judge whether the concerned (strongly) paired symmetric tensors are positive definite or not. Fourth, by using semidefinite relaxation we propose a sequential semidefinite programming method to compute the smallest M-eigenvalue of a fourth order three dimensional (strongly) paired symmetric tensor, by which we can check positive definiteness of the concerned tensor. The preliminary numerical results confirm our theoretical findings.

Key words. Paired symmetric tensor, elasticity tensor, positive definiteness of tensor, M-eigenvalue, semidefinite relaxation.

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