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Abstract

This paper deals with control strategies for discrete-time fractional multi–agent systems. By using the discrete fractional order operator we introduce memory effects to the considered problem. Necessary optimality conditions for discrete-time fractional optimal control problems with single- and double-integrator dynamics are proved, and simulation results are presented.

Keywords: multi–agent systems, emergent behavior, consensus, fractional-order systems, optimal control.
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1. Introduction

By an emergent collective behavior in multi-agent systems we mean interactions between individual agents that yield distinct patterns at the level of the group. This behavior cannot be understood simply as the sum of its constituent parts. Moreover, individual actions affect group outcomes, group outcomes feed back to affect individual actions. This behavior is widespread in the nature, for example in the form of flocking of birds, schooling of fish [2, 29], and its self-organization feature has attracted a lot of attention in engineering areas, especially in sensor networks [14, 16, 30]. Consequently, in the past few decades have seen significant research activity in applying mathematical tools to studying emergent collective behaviors in multi-agent systems. These include algebraic graph theory [22, 23, 24], dynamical systems theory [4, 7, 11, 17], time scale theory [9, 10, 32], control and optimal control theory [6, 33] and the fractional calculus theory [3, 18, 21, 31, 35] (the few examples cited here are a small sample from a vast literature).

In this paper we focus on discrete-time fractional multi–agent systems and investigate optimal control strategies for two variants of those type of systems. First, we consider a problem with single-integrator dynamics, that is each agent is described by time-dependent consensus parameter $x_i(t) \in \mathbb{R}$, $i = 1, \dots, N$, which could represent a certain physical/economic/social quantity such as attitude, position, temperature, price, opinion and so on. A multi–agent system is said to have reached consensus (or agreement) if and only if there exists $\bar{t}$ such that $x_i(t) = x_j(t)$ for all $i, j = 1, \dots, N$ and $t > \bar{t}$. In the situation when the system does not converge to a consensus or there is a demand to reach a consensus faster it is natural to use external control strategies. This control policy should be designed in the most economical way. Therefore, we minimize control and disagreements among agents. Next, we consider a problem with double-integrator dynamics, that is each agent is represented by two coordinates $(x_i(t), v_i(t)) \in \mathbb{R}^2$, where $x_i(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ are the time-dependent state and consensus parameter, respectively. Assuming that there is no interactions among agents we minimize in the time horizon control inputs together with the distance among agents and the discrepancy of the consensus variables to the mean of those variables. The motivation for using fractional operators is twofold. First, the integer-order systems are just

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