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# Optimal Uniform-Convergence Results for Convection-Diffusion Problems in One Dimension Using Preconditioning

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## Abstract

A linear one-dimensional convection-diffusion problem with a small singular perturbation parameter $\varepsilon$ is considered. The problem is discretized using finite-difference schemes on the Shishkin mesh. Generally speaking, such discretizations are not consistent uniformly in $\varepsilon$, so $\varepsilon$-uniform convergence cannot be proved by the classical approach based on $\varepsilon$-uniform stability and $\varepsilon$-uniform consistency. This is why previous proofs of convergence have introduced non-classical techniques (e.g., specially chosen barrier functions). In the present paper, we show for the first time that one can prove optimal convergence inside the classical framework: a suitable preconditioning of the discrete system is shown to yield a method that, uniformly in $\varepsilon$, is both consistent and stable. Using this technique, optimal error bounds are obtained for the upwind and hybrid finite-difference schemes.

*Keywords:* singular perturbation, convection-diffusion, Shishkin mesh, finite differences, uniform convergence, preconditioning
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