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A generalized fractional-order Legendre wavelet Tau method for solving fractional differential equations

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Abstract

A new fractional-order wavelet basis as generalization of the classical Legendre wavelet is defined. The operational matrices both for derivative and fractional derivative in the sense of Caputo for this fractional-order wavelet are derived. Then, a numerical scheme based on these operational matrices and the typical Tau method is proposed for solving some nonlinear fractional differential equations. Illustrative examples show that the present wavelet Tau method is numerically effective and convenient for solving fractional differential equations. Moreover, the obtained results confirm that, in comparison with the classical Legendre wavelet method, the fractional-order wavelet basis is more efficient and accurate for solving fractional differential equations. Error analysis and convergence of the proposed wavelet method is also provided.

Keywords: Fractional-order Legendre wavelets, Caputo derivative, Operational matrix, Tau method, Fractional differential equations

AMS Subject Classification: 26A33, 34A08, 34K37, 65T60

1 Introduction

Fractional calculus, as an extension of derivatives and integrals to non-integer orders, has been recently used to model many fundamental problems in Science and Engineering. For instance, fractional calculus was used for modelling the nonlinear oscillation of earthquake, fluid-dynamic traffic, continuum and statistical mechanics, signal processing, control theory, and dynamics of interfaces between nanoparticles and subtracts [1–3]. These models are based on various fractional differential equations, fractional partial differential equations, fractional integral equations, or in more complicated cases fractional integro-differential equations. In many cases analytic and exact solution of fractional differential and integral equations are not available. Consequently, the accurate numerical methods for solving these fractional equations is a challenging and motivational research field in mathematics and engineering. In the past decade, several numerical methods such as Laplace transforms method [1], Fourier transforms method [4], fractional differential transform method [5], finite difference method [6], eigenvector expansion [7], Adomian decomposition method [8], variational iteration method [9], homotopy perturbation method [10],