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The sub-Cauchy Stokes Problem: Solvability Issues and Lagrange Multiplier Methods with Artificial Boundary Conditions

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Abstract

In contrast to the conventional Cauchy Stokes problems, in which the velocity and the stress force data are given on the accessible boundary, in the present paper, we reduce the accessible boundary data information and we consider a problem which deals only with shear stress data. We refer to this problem as a sub-Cauchy Stokes problem. This problem is ill-posed because of severe instability and even uniqueness is unknown. We first address the uniqueness issues associated with this problem. Resorting to the domain decomposition techniques together with the duplication process of Vogelius [1], we propose new Lagrange multiplier methods to solve the sub-Cauchy Stokes problem. These methods consist in recasting the problem in terms of interfacial equations, by equalizing two solutions of the sub-Cauchy Stokes problem using matching conditions defined on the inaccessible boundary. The matching is based on second order conditions and the types of the interfacial equations depend on the equations used to match the values of the unknowns on the inaccessible boundary. The interfacial problems are then solved by iterative procedures in which coefficients can be optimized to improve convergence rates. A complete analysis of the methods is presented, and intensive numerical results illustrate the effectiveness and the performance of the proposed approaches.

Keywords: Inverse problem, Sub-Cauchy Stokes system, Uniqueness, Ventcell boundary conditions, Alternating method, Noise, Convergence factor.

1. Introduction

This work is concerned with the data completion problem associated with the Stokes system. This is a strongly ill-posed inverse problem. To describe the inverse problem, we let Ω be a bounded and simply connected domain of $\mathbb{R}^2$, with Lipschitz boundary $\Gamma = \partial\Omega$ made up of three connected disjoint open subsets satisfying $\cup_{i=\mathrm{I,D,C}}\overline{\Gamma_i} = \Gamma$. The portion Γ_{C} is considered as accessible to measures while Γ_{I} is the unreachable boundary and no data are thus available on it. Let us denote as usual, by $\mathbf{n}$ the outward unit normal vector and by $\boldsymbol{\tau}$ the unit tangent vector to the boundary Γ. For any vector field $\mathbf{v}$ on $\partial\Omega$, we shall denote by $\mathbf{v}_{\boldsymbol{\tau}}$ the projection of $\mathbf{v}$ on the tangent hyperplane to Γ. In other words $\mathbf{v}_{\boldsymbol{\tau}} = \mathbf{v} - (\mathbf{v}\cdot\mathbf{n})\mathbf{n}$.

The data completion problem we consider consists in finding the solution $(\mathbf{u}, p)$ of the incompressible Stokes equations

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega, \tag{1.1}$$

$$-\Delta\mathbf{u} + \nabla p = 0 \quad \text{in } \Omega, \tag{1.2}$$

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