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PII: S0377-0427(17)30461-2
DOI: https://doi.org/10.1016/j.cam.2017.09.030
Reference: CAM 11310

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 30 March 2017
Revised date : 19 August 2017

A practical formula of solutions for a family of linear non-autonomous fractional nabla difference equations

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Abstract. In this article, we focus on a generalised problem of linear non-autonomous fractional nabla difference equations. Firstly, we define the equations and describe how this family of problems covers other linear fractional difference equations that appear in the literature. Then, by using matrix theory we provide a new practical formula of solutions for these type of equations. Finally, numerical examples are given to justify our theory.

Keywords: non-autonomous, matrix, nabla, fractional, difference equations.

1 Introduction

Difference equations of fractional order have recently proven to be valuable tools in the modeling of many phenomena in various fields of science and engineering. Indeed, we can find numerous applications in viscoelasticity, electrochemistry, physics, control, porous media, electromagnetism and so forth, see [5], [6], [15], [20], [24], [25], [30], [32], [33]. At this point it is strongly believed that the fractional discrete operators can have important contribution in generalizing this idea to classical mechanics, non-relativistic quantum mechanics and relativistic quantum field theories.

The theory of discrete fractional equations is also a promising tool for several biological and physical applications where the memory effect appears. The dynamics of the complex systems are better described within this new powerful tool. The nanotechnology and its applications in biology for example as well as the discrete gravity are fields where the fractional discrete models will play an important role in the future, see [6], [15].

There has been a significant development in the study of fractional difference equations and inclusions in recent years; For some recent contributions focusing on the solutions of fractional difference equations, see [1], [2], [3], [4], [7], [8], [9], [10], [11], [14], [16], [17], [18], [21], [23], [27], [29], and the references therein. The stability of fractional difference equations has been studied in [12], [19], [22], [28], [31], [33], [34].

In this article we will use the fractional nabla operator as defined when applied to a sequence. The backward difference operator of first order, denoted by ∇ (nabla operator), when applied to a vector of sequences $Y_k : \mathbb{N} \to \mathbb{C}^m$ is defined by:

$$\nabla Y_k = Y_k - Y_{k-1};$$