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Robin Ahrens, Sabine Le Borne

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FFT-based evaluation of multivariate aggregation integrals in population balance equations on uniform tensor grids

Robin Ahrens[a], Sabine Le Borne[a,*]

[a] *Institute for Mathematics, Hamburg University of Technology, Am Schwarzenberg-Campus 3, 21073 Hamburg, Germany*

Abstract

We consider the numerical solution of the multivariate aggregation population balance equation on a uniform tensor grid. We introduce a multidimensional fast Fourier transformation for the efficient evaluation of the aggregation integrals leading to a reduction in the complexity order of the algorithm compared to the direct evaluation approach. We illustrate the new evaluation algorithm for two discretizations, an FEM approach as well as the sectional method. We discuss the conservation of moments for these methods and provide numerical comparisons illustrating the superior performance of FFT-based algorithms. We also discuss and numerically illustrate their potential for parallelization.

Keywords: Multivariate population balance equation; Aggregation; Convolution; FFT

1. Introduction

A particle population may be quantified by a number density function $f(\mathbf{x}, t)$ which describes the property distribution of the particles at given time t [1]. Here, $\mathbf{x} \in \mathbb{R}_+^d$ ($\mathbb{R}_+ := [0, \infty)$) denotes the particle state vector of $d \in \mathbb{N}$ additive properties like mass or concentration of some substance. Due to growth, birth and death phenomena, the density function f varies dynamically with time.

*Corresponding author, Tel.: +49 40 42878 3874; fax: +49 40 42878 2696;
Email address: leborne@tuhh.de (Sabine Le Borne)