Accepted Manuscript

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PII: S0377-0427(17)30415-6
DOI: http://dx.doi.org/10.1016/j.cam.2017.08.026
Reference: CAM 11278

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 18 April 2017
Revised date : 9 August 2017

Please cite this article as: E.F. Doungmo Goufo, J.J. Nieto, Attractors for fractional differential problems of transition to turbulent flows, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.08.026

Attractors for fractional differential problems of transition to turbulent flows

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Abstract

The complexity of fluid flows remains an intriguing problem and many scientists are still struggling to gain new and reliable insight into the dynamics of fluids. Transition from laminar to turbulent flows is even more complex and many of its features remain surprising and unexplained.
To describe transition to turbulence we introduce some fractional models and use numerical approximations to reveal the existence of attractor points. Two different cases are studied; the classical situation corresponding to the integer dimension one and the pure fractional case. The observed simulations show, in both cases, the presence of attractors near which iterations converge faster than usual. The behavior observed in the conventional case is in concordance with the well-know results that exist in the literature for relatively low order ordinary differential equations. The results observed in the fractional case are innovative since they reveal, not only the persistence of attractors, but also a possible better description of the transition to turbulent flows due to the variation of the fractional parameter that allows the control of the dynamics.

Keywords: Transition to turbulent flows; fractional dynamics, attractors, numerical approximations, stability, convergence

65L20; 33F05,76F06, 37D45, 26A33

1. Introduction to the model

The whole analysis conducted in this article consists of extending an initial value problem modeling the transition to turbulent flows in incompressible fluids. Hence, we fully investigate the existence and approximation of attractors for fractional differential equations (FDEs) of the particular form

$$D_t^{\alpha} y(t) = \|y\|^p K y(t) + B y(t) + g, \quad \alpha \in [0;1], \quad t \geq 0, \tag{1.1}$$

which are assumed to satisfy the initial condition

$$y(0) = \rho, \tag{1.2}$$