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Abstract

We address the study of well posedness on Lebesgue spaces of sequences for the following fractional semidiscrete model with finite delay

$$\Delta^\alpha u(n) = Tu(n) + \beta u(n-\tau) + f(n), \quad n \in \mathbb{N}, \ \ 0 < \alpha \leq 1, \ \ \beta \in \mathbb{R}, \ \ \tau \in \mathbb{N}_0, \tag{0.1}$$

where T is a bounded linear operator defined on a Banach space X (typically a space of functions like $L^p(\Omega), 1 < p < \infty$) and Δ^α corresponds to the time discretization of the continuous Riemann-Liouville fractional derivative by means of the Poisson distribution. We characterize the existence and uniqueness of solutions in vector-valued Lebesgue spaces of sequences of the model (0.1) in terms of boundedness of the operator-valued symbol

$$((z-1)^\alpha z^{1-\alpha} I - \beta z^{-\tau} - T)^{-1}, \quad |z| = 1, \ z \neq 1,$$

whenever $0 < \alpha \leq 1$ and X satisfies a geometrical condition. For this purpose, we use methods from operator-valued Fourier multipliers and resolvent operator families associated to the homogeneous problem. We apply this result to show a practical and computational criterion in the context of Hilbert spaces.

Keywords: Fractional differences; Delay equations; Well-posedness; Maximal regularity; Operator-valued Fourier multiplier.

1. Introduction

Time delay equations appear in computational and applied contexts that have been the subject of research of many authors in the last decades. For instance, Fu and Li in [17], studied the well posedness of evolution equations with infinite delay in Lebesgue, Besov and Triebel-Lizorkin scales of vector-valued Banach spaces. Other references in this line of research are [8], [13], [15], [29] and [30]. See also the bibliography therein. In the semidiscrete case, there is also a big amount of investigation where the study of such equations naturally arises. In a first survey by H. Bateman [7] their relevance in several practical fields of interest was explained. Some of such applications appear in the theory of the compound pendulum, surges in springs and connected systems of springs, equations of Born and von Kármán and

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