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Yin Yang, Yunqing Huang, Yong Zhou

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NUMERICAL SOLUTIONS FOR SOLVING TIME FRACTIONAL FOKKER-PLANK EQUATIONS BASED ON SPECTRAL COLLOCATION METHODS

YIN YANG[1,*], YUNQING HUANG[1], AND YONG ZHOU[1,*]

Abstract. In this paper, we consider the numerical solution of the time fractional Fokker-Plank equation. We propose a spectral collocation method in both temporal and spatial discretizations with a spectral expansion of Jacobi interpolation polynomial for this equation. The convergence of the method is rigourously established. Numerical tests are carried out to confirm the theoretical results.

keywords: fractional Fokker-Plank equation; Jacobi collocation method; convergence.
AMS subject classifications: 35R35, 65M12, 65M70

1. Introduction

The time evolution of the probability density function of position and velocity of a particle, which is one of the classical, widely used equations of statistical physics was modeled many times based on the linear Smolochowski Fokker-Planck equation (FPE)

$$\frac{\partial}{\partial t}P(z,t)=\left[\frac{\partial}{\partial z}\frac{V'(z)}{m\eta_1}+K_1\frac{\partial^2}{\partial z^2}\right]P(z,t), \tag{1.1}$$

which defines the probability density function $P(z,t)$ to find the test particle at a certain position z at a given time t. where $V(z)$ indicates the potential of overdamped Brownian motion, a prime stands for the derivative w.r.t. the space coordinate z, m denotes the mass of the particle, K_1 denotes the diffusion constant associated with the transport process, and the friction coefficient η_1 is a measure for the interaction of the particle with its environment. The basic properties of the FPE are the exponential decay of the modes, the Einstein relations which are intimately connected with the fluctuation-dissipation theorem and with linear response, and the Gaussian evolution in the force-free case. For example, in the force-free case, i.e., $V(z)=$ const, the corresponding diffusion process is governed by Ficks second law, leading to the linear time dependence

$$< z^2(t) >= 2K_1 t \tag{1.2}$$

of the mean square displacement; this hallmark of Gaussian diffusion is a consequence of the central limit theorem. In a variety of systems one find that the (1.2) is violated.

[1]School of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, Hunan, P.R.China.

*Corresponding author. E-mail: yzhou@xtu.edu.cn (Yong Zhou), yangyinxtu@xtu.edu.cn (Yin Yang).