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Oscillation analysis of numerical solutions for delay differential equations with real coefficients[☆]

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ABSTRACT

The paper is mainly concerned with the oscillations of numerical solutions by the linear $\theta-$methods for a kind of linear delay differential equations with positive and negative coefficients. Some conditions under which the numerical solutions are oscillatory are obtained and it is proved that oscillations of the analytical solutions are preserved by the numerical solutions under mild conditions. Numerical experiments are given to demonstrate our results.

1. Introduction

In recent years, there has been much research onto the oscillation theory of delay differential equations. To a large extent, this is due to the fact that delay differential equations are widely applied in physics, ecology, biology, see [1–10]. There is no doubt that some of the recent developments in oscillation theory have contributed a beautiful body of knowledge in the field of differential equations that have enhanced our understanding of the qualitative behavior of their solutions. But these papers are most concerning on oscillations of analytical solutions, only a few papers have been devoted to oscillation theory of numerical solutions for delay differential equations. It is easy to see that oscillation theory of numerical solutions will assume greater importance in the industrial field. Therefore, the various researches about oscillation of numerical solutions for delay differential equation are increasing. Papers [11–13] are concerned with the preservation of oscillations for delay differential equations with piecewise constant arguments. References [14–16] provide an interesting method for the research of numerical oscillation for nonlinear delay differential equation and obtain the conditions for oscillation of numerical solutions under some conditions. But the coefficients for these equations are all positive or negative. In fact, most of the models used in vital statistics involve birth and death rates and thus differential equations with positive and negative coefficients are of crucial importance. It is worth mentioning that there is no paper concerning with the oscillation of the numerical solutions for delay differential equations with positive and negative coefficients as far as we know. Hence, in this paper, we will consider the oscillation in the θ-method for the following equation

$$x'(t) + \sum_{i=1}^{l} p_i x(t-\tau_i) - \sum_{j=1}^{r} q_j x(t-\sigma_j) = 0, \tag{1.1}$$

where

$$p_i,\ q_j,\ \tau_i,\ \sigma_j \in \mathbb{R}^+,\ i = 1, 2, \ldots, l,\ j = 1, 2, \ldots, r \tag{1.2}$$

$$\textit{with } \tau_1 \geq \tau_2 \geq \cdots \geq \tau_l \textit{ and } \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r.$$

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The outline of the paper is as follows. In Section 2, we give the preliminaries. In Section 3, the oscillations of the numerical solutions will be discussed and the sufficient conditions under which the numerical solutions oscillate are obtained. The numerical experiments will be given in Section 4.

2. Preliminaries

In this section, we will give the definitions of oscillations and some conditions for oscillations.

Definition 2.1 ([17]). A non-constant solution $x(t)$ of Eq. (1.1) is said to oscillate about K if $x(t)-K$ has arbitrarily large zeros. Otherwise, $x(t)$ is non-oscillatory. When $K=0$, we say that $x(t)$ oscillates about zero or simply oscillates. If $x(t)\equiv K$, we also say $x(t)$ is oscillatory about K.

Definition 2.2 ([17]). We say Eq. (1.1) oscillates if all of its solutions are oscillatory.

Theorem 2.3 ([18]). *Assume that*

$$lp_i > \sum_{j=1}^{r} q_j, \quad \tau_i \geq \sigma_1 \text{ for } i = 1, 2, \ldots, l, \tag{2.1}$$

$$\sum_{i=1}^{l}\Big(1 - \sum_{j=1}^{r} q_j(\tau_i - \sigma_j)\Big) \geq 0 \tag{2.2}$$

and

$$\sum_{i=1}^{l}\Big(lp_i - \sum_{j=1}^{r} q_j\Big)\tau_i > \frac{1}{e}\Big(\sum_{i=1}^{l}(1 - \sum_{j=1}^{r} q_j(\tau_i - \sigma_j))\Big), \tag{2.3}$$

then Eq. (1.1) oscillates.

Corollary 2.4. *If (2.1), (2.2) and*

$$\sum_{i=1}^{l}\Big(lp_i - \sum_{j=1}^{r} q_j\Big)\tau_l > \frac{1}{e}\Big(\sum_{i=1}^{l}(1 - \sum_{j=1}^{r} q_j(\tau_i - \sigma_j))\Big) \tag{2.4}$$

hold, then Eq. (1.1) oscillates.

Similar to Definition 2.1, we give the definition of oscillations for a sequence of real numbers as follows.

Definition 2.5 ([17]). A non-constant sequence of real numbers $\{x_n\}$ is said to oscillate about $\{a_n\}$ if $\{x_n\}-\{a_n\}$ is neither eventually positive nor eventually negative. Otherwise, $\{x_n\}$ is non-oscillatory. If $\{a_n\}=\{a\}$ is a constant sequence, we simply say that $\{x_n\}$ oscillates about a. When $\{a_n\}=0$, we say that $\{x_n\}$ oscillates about zero or simply oscillates. If $\{x_n\}\equiv a$, we also say $\{x_n\}$ is oscillatory about a.

The following results are vital for our analysis of numerical oscillations.

Theorem 2.6 ([17]). *Consider the difference equation*

$$a_{n+1} - a_n + \sum_{j=-k}^{l} q_j a_{n+j} = 0. \tag{2.5}$$

Assume that $k, l \in \mathbb{N}$ and $q_j \in \mathbb{R}$ for $j=-k,\ldots,l$. Then the following statements are equivalent:

(i) *Every solution of Eq. (2.5) oscillates.*
(ii) *The characteristic equation $\lambda - 1 + \sum_{j=-k}^{l} q_j\lambda^j = 0$ has no positive roots.*

Theorem 2.7 ([17]). *Consider the equation*

$$y_{n+1} - y_n + \sum_{i=1}^{m} p_i y_{n-k_i} = 0, \ n = 0, 1, 2, \ldots, \tag{2.6}$$

assume that $p_i > 0$, $k_i \in \mathbb{N}^+$ for $i=1,2,\ldots,m$, and

$$\sum_{i=1}^{m} p_i \frac{(k_i+1)^{k_i+1}}{k_i^{k_i}} > 1,$$

then every solution of Eq. (2.6) oscillates.