Accepted Manuscript

Preserving the order of convergence: Low-complexity Jacobian-free iterative schemes for solving nonlinear systems

A.R. Amiri, A. Cordero, M.T. Darvishi, J.R. Torregrosa

PII: S0377-0427(18)30025-6
DOI: https://doi.org/10.1016/j.cam.2018.01.004
Reference: CAM 11476

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 11 October 2017
Revised date : 29 December 2017

Preserving the order of convergence: low-complexity Jacobian-free iterative schemes for solving nonlinear systems ☆

A. R. Amiri[a], A. Cordero[b,*], M. T. Darvishi[a], J. R. Torregrosa[b]

[a] *Department of Mathematics, Razi University, Iran*
[b] *Instituto de Matemáticas Multidisciplinar, Universitat Politècnica de València, Spain*

Abstract

In this paper, a new technique to construct a family of divided differences for designing derivative-free iterative methods for solving nonlinear systems is proposed. By using these divided differences any kind of iterative methods containing a Jacobian matrix in its iterative expression can be transformed into a "Jacobian-free" scheme preserving the order of convergence. This procedure is applied on different schemes, showing theoretically their order and error equation. Numerical experiments confirm the theoretical results and show the efficiency and performance of the new Jacobian-free schemes.

Keywords: Nonlinear system of equations, iterative method, Jacobian-free scheme, divided difference, order of convergence.

1. Introduction

Nonlinear systems are of interest to engineers, physicists, mathematicians and other scientists, because the modelization of many nonlinear problems arising in different fields of science is made by means of a nonlinear system of equations.

Let $F(x) = 0$ be a system of nonlinear equations, where $F : D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}^n$ and f_i, $i = 1, 2, \ldots, n$, are the coordinate functions of F, $F(x) = (f_1(x), f_2(x), \ldots, f_n(x))^T$. Nonlinear systems are difficult to solve, the solution $\overline{x}$ usually is obtained by linearizing the nonlinear problem or using a fixed point function $G : D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}^n$, which leads to a fixed point iteration scheme. There are many finding-root methods for systems of nonlinear equations. The most famous one is the second order Newton method,

$$x^{(k+1)} = x^{(k)} - \left[F'(x^{(k)})\right]^{-1} F(x^{(k)}), \qquad (1)$$

where $F'(x^{(k)})$ is the Jacobian matrix of F evaluated at kth iteration.

In recent decades, many authors have tried to design iterative procedures with better efficiency and higher order of convergence than Newton's scheme (see, for example, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and the references therein). Most of them need to evaluate the Jacobian matrix at one or more points per iteration. One of the difficulties of using these methods is the computation of the Jacobian matrix, that in some cases, may not exist, or when it exists, for high dimensional cases, computing the Jacobian matrix is too costly or even in some cases unviable. Therefore, some authors have tried to omit the Jacobian matrix, replacing it by a divided difference operator. The simplest one is Steffensen's scheme [11], obtained by replacing the Jacobian matrix in Newton's method by a first-order divided difference, preserving the second order of convergence,

$$x^{(k+1)} = x^{(k)} - [z^{(k)}, x^{(k)}; F]^{-1} F(x^{(k)}),$$

☆This research was partially supported by Ministerio de Economía y Competitividad MTM2014-52016-C2-2-P, MTM2015-64013-P and Generalitat Valenciana PROMETEO/2016/089.

*Corresponding author
Email address: acordero@mat.upv.es (A. Cordero)