

## Accepted Manuscript

An efficient numerical method for a two-point boundary value problem with a Caputo fractional derivative

Zhongdi Cen, Jian Huang, Aimin Xu

PII: S0377-0427(17)30638-6  
DOI: <https://doi.org/10.1016/j.cam.2017.12.018>  
Reference: CAM 11435

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 30 August 2017  
Revised date: 13 November 2017

Please cite this article as: Z. Cen, J. Huang, A. Xu, An efficient numerical method for a two-point boundary value problem with a Caputo fractional derivative, *Journal of Computational and Applied Mathematics* (2017), <https://doi.org/10.1016/j.cam.2017.12.018>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# An efficient numerical method for a two-point boundary value problem with a Caputo fractional derivative

Zhongdi Cen, Jian Huang\* , Aimin Xu

Institute of Mathematics, Zhejiang Wanli University, Ningbo, China

**Abstract:** In this paper a two-point boundary value problem with a Caputo fractional derivative is considered. By using a shooting method based on the secant iterative method, the boundary value problem is turned into an initial value problem. Then the initial value problem is transformed into an equivalent integral-differential equation with a weakly singular kernel. An integral discretization scheme on the uniform mesh is developed to approximate the integral-differential equation. By applying the truncation error estimate techniques and a discrete analogue of Gronwall's inequality, it is proved that the numerical scheme is first-order convergent in the discrete maximum norm. Numerical experiments verify the theoretical results.

**Keywords:** Caputo fractional derivative; boundary value problem; shooting method; integral-differential equation; convergence analysis

**AMS subject classifications:** 65L05; 65L12; 65L20

## 1 Introduction

Fractional order differential equations, as generalizations of classical integer order differential equations, have been proved to be a valuable tool in the modelling of many phenomena, such as finance [13, 15], physics [8, 18], chemistry [10] and biology [6]. The analytical solutions of most fractional differential equations can not be obtained, so approximate and numerical techniques must be used.

In this paper we consider the following two-point boundary value problem with a Caputo fractional derivative

$$-D_*^\delta u(x) - (bu)'(x) + c(x)u(x) = f(x), \quad x \in (0, 1), \quad (1.1)$$

$$u(0) - \alpha_0 u'(0) = \gamma_0, \quad u(1) + \alpha_1 u'(1) = \gamma_1, \quad (1.2)$$

where  $D_*^\delta$  denotes the Caputo fractional derivative defined by

$$D_*^\delta u(x) = \frac{1}{\Gamma(2-\delta)} \int_0^x (x-t)^{1-\delta} u''(t) dt, \quad 1 < \delta < 2,$$

the constants  $\alpha_0, \alpha_1, \gamma_0, \gamma_1$  and the functions  $b, c$  and  $f$  are given. We assume that

$$b \geq \eta > 0, \quad c \leq \lambda, \quad c - b' \geq 0 \quad \text{for } x \in [0, 1] \quad \text{and} \quad \alpha_0 \geq \frac{1}{\delta-1}, \quad \alpha_1 \geq 0, \quad (1.3)$$

where  $\eta$  and  $\lambda$  are constants. These hypotheses guarantee that problem (1.1)-(1.2) satisfies a maximum principle, from which existence and uniqueness of the solution  $u$  of problem (1.1)-(1.2) is established in [17] by using the space  $C^{q,\nu}(0, 1]$ , where  $C^{q,\nu}(0, 1]$  denotes the space of

---

\*Corresponding author. Email: sword@zwu.edu.cn (Jian Huang).

Download English Version:

<https://daneshyari.com/en/article/8902067>

Download Persian Version:

<https://daneshyari.com/article/8902067>

[Daneshyari.com](https://daneshyari.com)