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An efficient numerical method for a two-point boundary value problem with a Caputo fractional derivative

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Abstract: In this paper a two-point boundary value problem with a Caputo fractional derivative is considered. By using a shooting method based on the secant iterative method, the boundary value problem is turned into an initial value problem. Then the initial value problem is transformed into an equivalent integral-differential equation with a weakly singular kernel. An integral discretization scheme on the uniform mesh is developed to approximate the integral-differential equation. By applying the truncation error estimate techniques and a discrete analogue of Gronwall's inequality, it is proved that the numerical scheme is first-order convergent in the discrete maximum norm. Numerical experiments verify the theoretical results.

Keywords: Caputo fractional derivative; boundary value problem; shooting method; integral-differential equation; convergence analysis

AMS subject classifications: 65L05; 65L12; 65L20

1 Introduction

Fractional order differential equations, as generalizations of classical integer order differential equations, have been proved to be a valuable tool in the modelling of many phenomena, such as finance [13, 15], physics [8, 18], chemistry [10] and biology [6]. The analytical solutions of most fractional differential equations can not be obtained, so approximate and numerical techniques must be used.

In this paper we consider the following two-point boundary value problem with a Caputo fractional derivative

$$-D_*^{\delta}u(x) - (bu)'(x) + c(x)u(x) = f(x), \qquad x \in (0,1), \tag{1.1}$$

$$u(0) - \alpha_0 u'(0) = \gamma_0, \quad u(1) + \alpha_1 u'(1) = \gamma_1, \tag{1.2}$$

where D_*^{δ} denotes the Caputo fractional derivative defined by

$$D_*^{\delta}u(x) = \frac{1}{\Gamma(2-\delta)}\int_0^x (x-t)^{1-\delta}\, u''(t)\mathrm{d}t, \qquad 1 < \delta < 2,$$

the constants $\alpha_0, \alpha_1, \gamma_0, \gamma_1$ and the functions b, c and f are given. We assume that

$$b \geq \eta > 0, \quad c \leq \lambda, \quad c - b' \geq 0 \quad \text{for} \quad x \in [0,1] \quad \text{and} \quad \alpha_0 \geq \frac{1}{\delta - 1}, \quad \alpha_1 \geq 0, \tag{1.3}$$

where η and λ are constants. These hypotheses guarantee that problem (1.1)-(1.2) satisfies a maximum principle, from which existence and uniqueness of the solution u of problem (1.1)-(1.2) is established in [17] by using the space $C^{q,\nu}(0,1]$, where $C^{q,\nu}(0,1]$ denotes the space of

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