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Analysis of a stabilized finite element method for Stokes equations of velocity boundary condition and of pressure boundary condition

Shuaishuai Du* and Huoyuan Duan*

Abstract

In this paper, a new pressure stabilized finite element method is analyzed for solving the Stokes equations. The key feature of the method is using the curl integral instead of the standard Dirichlet integral and using the pressure stabilization instead of the inf-sup condition. Thus, the method is very flexibly applicable to two problems of Stokes equations in terms of velocity and pressure with either pressure-Dirichlet boundary condition or velocity-Dirichlet boundary condition. For both Stokes problems, the finite element scheme and the finite element space of velocity are the same and one, where only the finite element spaces of pressure have small differences. A general analysis of stability and error estimates is developed and applications to both Stokes problems are further analyzed. The method covers the low regularity solution of very weak non H^1 space solution of Stokes problem of pressure-Dirichlet boundary condition as well as higher regularity solutions of Stokes problems of either pressure-Dirichlet boundary condition or velocity-Dirichlet boundary condition. Numerical results are presented to illustrate the performance of the method and to confirm the theoretical results.

Keywords Stokes equations, velocity Dirichlet boundary condition, pressure Dirichlet boundary condition, pressure stabilized finite element method, stability, error estimates.

1 Introduction

Let $\Omega \subset R^d (d = 2, 3)$ be a simply-connected bounded polygonal or polyhedral domain with a connected Lipschitz-continuous polygonal boundary Γ. The Stokes equations are as follows:

$$\begin{aligned} -\Delta \mathbf{u} + \nabla p &= \mathbf{f} \quad \text{in } \Omega, \\ \operatorname{div} \mathbf{u} &= 0 \quad \text{in } \Omega, \end{aligned} \tag{1.1}$$

where $\mathbf{u}$ is velocity and p pressure, and $\mathbf{f}$ is the volume force. There are many possible boundary conditions for $\mathbf{u}$ and p as the supplement of the well-posedness of the Stokes equations. We are concerned about the following two boundary conditions.

(VDB) Velocity Dirichlet boundary condition:

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma, \tag{1.2}$$

while pressure p is unique up to a constant and is asked to satisfy

$$\int_\Omega p = 0. \tag{1.3}$$

(PDB) Pressure Dirichlet boundary condition:

$$p = 0 \quad \text{on } \Gamma, \tag{1.4}$$

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