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# A block-centered finite difference method for an unsteady asymptotic coupled model in fractured media aquifer system *

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**Abstract.** A block-centered finite difference method is proposed for solving an unsteady asymptotic coupled model, in which the flow is governed by Darcy's law both in the one-dimensional fracture and two-dimensional porous media. The second-order error estimates in discrete norms are derived on nonuniform rectangular grids for both pressure and velocity. The numerical scheme can be extended to nonmatching spatial and temporal grids without loss of accuracy. Numerical experiments are performed to verify the efficiency and accuracy of the proposed method. It is shown that the pressure and velocity are discontinuous across the fracture-interface and the fracture indeed acts as the a fast pathway or geological barrier in the aquifer system.

**Keywords:** Karst aquifers; block-centered finite difference method; asymptotic coupled model.

## 1 Introduction

In Karst aquifer system, the fracture is considered to be the storage space of drinkable groundwater and occurrence site of environmental pollution. It is important to gain a better understanding of groundwater flow in Karst aquifer with fractures, in order to assess groundwater risk and control groundwater pollution. Due to the close connection between the fracture and surrounding medium, a coupled model is usually applied to demonstrate the groundwater flow process in fractured media aquifer system, where the flux exchange occurring on the interface between the fracture and surrounding medium is treated as the coupling term (see [1, 2, 3, 4] for details). The coupled model is given as follows:

$$
\begin{cases}
s_i\partial_t p_i + \mathrm{div}\mathbf{u}_i = g_i, & \text{in } \Omega_i \times J, \quad i = 1,2,f, & (1.1)\\
\mathbf{u}_i = -\mathbb{K}_i \nabla p_i, & \text{in } \Omega_i \times J, \quad i = 1,2,f, & (1.2)\\
\mathbf{u}_i\cdot\mathbf{n}_i = \mathbf{u}_f\cdot\mathbf{n}_i, & \text{on } \gamma_i \times J, \quad i = 1,2, & (1.3)\\
p_i = p_f, & \text{on } \gamma_i \times J, \quad i = 1,2, & (1.4)\\
\mathbf{u}_i\cdot\nu_i = v_i, & \text{on } \Gamma_i \times J, \quad i = 1,2,f, & (1.5)\\
p_i(\cdot,0) = q_i^0, & \text{in } \Omega_i, \quad i = 1,2,f, & (1.6)
\end{cases}
$$

where $\Omega_f$ is the region of fracture, $\Omega_1$ and $\Omega_2$ are the domains of porous media divided by the fracture $\Omega_f$, $\gamma_i = (\partial\Omega_i \cap \partial\Omega_f)\cap\Omega$, $\Gamma_i = \partial\Omega_i \cap \partial\Omega$, $\nu_i$ is unit outer normal direction to $\Omega_i$ $(i = 1,2,f)$, the time interval $J = (0,T]$, $\mathbf{u}$ is the Darcy velocity, $p$ is the pressure, $s$ is the storage coefficient,

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