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ADAPTIVE PIECEWISE TENSOR PRODUCT WAVELETS SCHEME FOR LAPLACE-INTERFACE PROBLEMS

NABI CHEGINI AND ROB STEVENSON

ABSTRACT. A Laplace type boundary value problem is considered with a generally discontinuous diffusion coefficient. A domain decomposition technique is used to construct a piecewise tensor product wavelet basis that, when normalised w.r.t. the energy-norm, has Riesz constants that are bounded uniformly in the jumps. An adaptive wavelet Galerkin method is applied to solve the boundary value problem with the best nonlinear approximation rate from the basis, in linear computational complexity. Although the solutions are far from smooth, numerical experiments in two dimensions show rates as for a one-dimensional smooth solution, the latter being possible because of the tensor product construction.

1. INTRODUCTION

In this paper, we study second order linear elliptic problems, generally with a discontinuous diffusion coefficient, that are known as *Laplace-interface problems* or *transmission problems*. For some domain $\Omega \subset \mathbb{R}^n$, $\Gamma \subset \partial\Omega$ with $|\Gamma| > 0$, and given $f \in H^1_{0,\Gamma}(\Omega)'$, we consider the problem of finding $u \in H^1_{0,\Gamma}(\Omega)$ such that

$$a_\kappa(u,v) := \int_\Omega \kappa \nabla u \cdot \nabla v = f(v) \quad \forall v \in H^1_{0,\Gamma}(\Omega). \tag{1.1}$$

For some *fixed* N, and $0 \leq i \leq N$, let $\Omega_i \subset \Omega$ be mutually disjoint hypercubes such that $\bar{\Omega} = \cup_{i=0}^N \bar{\Omega}_i$. We assume that

$$\kappa|_{\Omega_i} = \kappa_i, \quad i = 0, \cdots, N, \tag{1.2}$$

where each κ_i is a positive constant, and that Γ is the closure of the union of facets of one or more Ω_i. The coefficient in problem (1.1) may have large jumps across interfaces between the hypercubes. Consequently, the solution can be expected to be non-smooth at these interfaces, in particular in directions normal to them. It is known (see [Kel74]) that for $\Omega = (-1,1)^2$ partitioned into four unit squares, for any $\varepsilon > 0$, there exists a κ such that, for smooth f, the solution of (1.1) is not in $H^{1+\varepsilon}(\Omega)$.

Because of the non-smoothness of the solution, we solve the problem numerically with an adaptive method, where we take the *Adaptive Wavelet-Galerkin Method* (**AWGM**) ([CDD01, GHS07, Ste09]). To do so, we equip $H^1_{0,\Gamma}(\Omega)$ with a Riesz basis $\boldsymbol{\Psi} = \{\boldsymbol{\psi}_\lambda : \boldsymbol{\lambda} \in \boldsymbol{\nabla}\}$ that has Riesz constants that are bounded *uniformly*

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