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PARALLEL CHARACTERISTIC MIXED ELEMENT METHOD FOR SALTWATER INTRUSION PROBLEM

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Abstract. Numerical simulation of saltwater intrusion problem has attracted more and more attention in the past decades. Based on parallel subspace correction method proposed by J. Xu in [31], a new overlapping domain decomposition parallel algorithm is constructed to solve saltwater intrusion problem. In this algorithm, the splitting positive definite mixed element method is used to discrete head water equation, while the characteristic Galerkin finite element method is used to approximate concentration equation. The relation between the convergence rate and discretization parameters, including the overlapping degree of the subspaces, is considered. Furthermore, the convergence of this algorithm is analyzed and the corresponding error estimate is given. The convergence result suggests that only two iterations are needed to reach the given accuracy at each time level.

1. Introduction

In recent years, saltwater intrusion has occurred in many countries and regions all over the world, and caused great damages to industrial and agricultural productions, it is urgent to be tackled. With the increasing interest, there are more and more literatures on this problem in past decades, see [1, 2, 3, 4, 5, 6]. A lot of work on numerical methods for this problem have been done, for example, characteristic finite difference methods [7], characteristic finite element methods [8, 9], upwind fractional-step finite difference methods [10, 11, 12, 13] and alternating-direction methods [14, 15]. To obtain more accurate approximation of the flux function, the classical mixed finite element method was used in [16]. But the technique of the classical mixed finite element method leads to some saddle point problem whose numerical solutions have been quite difficult because of losing positive definite properties. To overcome this shortcoming, the authors of this article studied the splitting positive definite mixed finite element method (see [17, 18, 19, 20, 21, 22, 23]), in which the mixed system is symmetric positive definite and decoupled.

As we know, saltwater intrusion problem refers to a large-scale, complex boundary problem. Domain decomposition parallel algorithm is a powerful tool to deal with this kind of problems. There are many literatures on overlapping domain decomposition parallel algorithms for elliptic problems (see [29, 30, 31, 32]) and parabolic problems (see [24, 25, 26, 27, 28, 33, 34, 35, 36, 37]). However, to our knowledge, there are few works on parallel algorithm for saltwater intrusion problem. So, it is very important and useful to develop effective parallel algorithms for saltwater intrusion problem both in engineering applications and mathematical analysis.

In this paper, based on overlapping domain decomposition, a new parallel algorithm is established for solving saltwater intrusion problem with characteristic splitting mixed finite element procedure (see [22]). The development of the method is carried out by using a parallel subspace correction framework and by utilizing the partition functions of unity to distribute the corrections in the overlapping domains reasonably. The relation between the convergence rate and discretization parameters is considered, and the corresponding error estimate is given. The convergence result suggests that only two iterations are needed to reach the given accuracy at each time level.

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