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CONDITIONING AND ERROR ANALYSIS OF NONLOCAL OPERATORS WITH LOCAL BOUNDARY CONDITIONS

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Abstract. We study the conditioning and error analysis of novel nonlocal operators in 1D with local boundary conditions. These operators are used, for instance, in peridynamics (PD) and nonlocal diffusion. The original PD operator uses nonlocal boundary conditions (BC). The novel operators agree with the original PD operator in the bulk of the domain and simultaneously enforce local periodic, antiperiodic, Neumann, or Dirichlet BC. We prove sharp bounds for their condition numbers in the parameter δ only, the size of nonlocality. We accomplish sharpness both rigorously and numerically. We also present an error analysis in which we use the the Nyström method with the trapezoidal rule for discretization. Using the sharp bounds, we prove that the error bound scales like $\mathcal{O}(h^2\delta^{-2})$ and verify the bound numerically.

The conditioning analysis of the original PD operator was studied by Aksoylu and Unlu (SIAM J. Numer. Anal. 52:653–677, 2014). For that operator, we had to resort to a discretized form because we did not have access to the eigenvalues of the analytic operator. Due to analytical construction, we now have direct access to the explicit expression of the eigenvalues of the novel operators in terms of δ. This gives us a big advantage in finding sharp bounds for the condition number without going to a discretized form and makes our analysis easily accessible. We prove that the novel operators have ill-conditioning indicated by δ^{-2} sharp bounds. For the original PD operator, we had proved the similar δ^{-2} ill-conditioning when the mesh size approaches 0. From the conditioning perspective, we conclude that the modification made to the original PD operator to obtain the novel operators that accommodate local BC is minor. Furthermore, the sharp δ^{-2} bounds shed light on the role of δ in nonlocal problems.

Keywords: Condition number, error analysis, integral operator, peridynamics, nonlocal diffusion, preconditioning.
Mathematics Subject Classification (2000): 65F35, 47G10, 74B99.

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