Structured condition numbers for some matrix factorizations of structured matrices

Hanyu Li

PII: S0377-0427(17)30630-1
DOI: https://doi.org/10.1016/j.cam.2017.12.010
Reference: CAM 11427

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 28 August 2016
Revised date : 9 October 2017

Please cite this article as: H. Li, Structured condition numbers for some matrix factorizations of structured matrices, *Journal of Computational and Applied Mathematics* (2017), https://doi.org/10.1016/j.cam.2017.12.010

Structured condition numbers for some matrix factorizations of structured matrices*

Hanyu Li†

College of Mathematics and Statistics, Chongqing University, Chongqing, 401331, P. R. China

Abstract

Using the modified matrix-vector approach and the differential calculus, we study the structured condition numbers for LU, Cholesky and QR factorizations of some structured matrices that can be represented by sets of parameters. The obtained explicit expressions of these structured condition numbers are very general, which are applicable to most of linear and non-linear structured matrices, and include the popular normwise, mixed and componentwise condition numbers as special cases. More specific explicit expressions of the structured condition numbers for linear structured matrices are also provided. We compare the structured condition numbers with the corresponding unstructured ones in theory and experiment. Numerical results show that, for non-linear structured matrices, the structured condition numbers can be much smaller than the unstructured ones. In addition, we also test the applications of structured condition numbers in estimating the first-order perturbation bounds of matrix factorizations using numerical examples.
AMS classification: 65F35; 15A23; 15A57

Keywords: LU factorization; Cholesky factorization; QR factorization; Structured condition number; Normwise condition number; Mixed condition number; Componentwise condition number; Structured matrix

1 Introduction

It is well-known that LU, Cholesky, and QR factorizations are the basic and effective tools in numerical linear algebra (see e.g., [1, 2]). Many scholars investigate their applications, algorithms, stability of algorithms, and perturbation analysis (see e.g., [1, 2, 3, 4, 5, 6, 7]). On the other hand, it is also well-known that condition number plays an important role in numerical algorithm. For example, combining the condition number and the backward error of a backward stable algorithm, we can estimate the forward error committed by the algorithm. So, many works on condition number have been published; see the recent book on condition [8].

Putting the above two topics together, i.e., investigating the condition numbers for the above three matrix factorizations, arouses the scholars' interest. Some work has been done on this subject. Chang et al. [9, 10, 11] first obtained the normwise condition numbers for LU and Cholesky factorizations and for the factor R of QR factorization. However, the obtained expressions cannot clearly emerge the relationship between them and the factors of matrix factorizations and hence are not explicit. Since the normwise condition number cannot accurately reflect the influence of perturbations for some small entries in the data and ignores the structures of input and output data with respect to scaling, Wang and Wei [12, 13] investigated the mixed and componentwise condition numbers for the above three matrix factorizations and presented their explicit expressions. These two condition numbers were named by Gohberg

*The work is supported by the National Natural Science Foundation of China (Grant No. 11671060).
†Email addresses: lihy.hy@gmail.com or hyli@cqu.edu.cn