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On the rediscovery of Halley's iterative method for computing the zero of an analytic function

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Abstract

We show that Halley's basic sequence, resulting from accelerating the order of convergence of Newton's method, is the most efficient way of doing so in terms of usage of certain derivatives. This fact could explain why this process of accelerating the convergence of Newton's method is so frequently rediscovered. Then we present an algorithmic way of recognizing Halley's family and we apply this algorithm to examples of rediscoveries.

Keywords: Newton's method; Halley's method; high-order method; Taylor's expansion.
2010 MSC: 65-01; 65B99; 65H05.

1. Introduction

Newton's method of order 2 to solve a nonlinear equation appeared in 1669 [26]. Later, in 1694, Halley presented an improvement of order 3 of this method [6, 4]. In 1870 Schröder introduced an infinite sequence of methods based on rational approximations whose p^{th} member is of order p. The first two elements of this sequence where Newton's method ($p = 2$), and Halley's method ($p = 3$). This family is said to be the Schröder's method of 2nd kind, or also the Halley's basic sequence. This sequence, under different forms, has been rediscovered in 1946 [7], and 1953 [8]. In [22] Traub says that Halley's method has been very often rediscovered. That statement was made in 1964, since then, this sequence has been rediscovered several times: 1966 [24], 1969