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Parameterized approximate block LU preconditioners for generalized saddle point problems[☆]

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Abstract

In this paper, we are concerned with the iteration solution of generalized saddle point problems. Based on the exact block LU factorization of the coefficient matrix, we construct a class of parameterized approximate block LU factorization preconditioners, which rely on suitable approximations of the Schur complement of the (1,1) block of the coefficient matrix. Convergence of the corresponding iteration methods are analyzed and the optimal iteration parameters minimizing the spectral radii are deduced. Algebraic characteristics of the related preconditioned matrices are discussed, including eigenvalue and eigenvector distributions and upper bounds for degree of the minimal polynomial. The established results extend those of the approximate factorization and variants of the Hermitian and skew-Hermitian splitting and positive and skew-Hermitian splitting preconditioners for saddle point problems. Numerical experiments are demonstrated to illustrate the efficiency of the new preconditioners.

Key words:
Block LU factorization, preconditioning, generalized saddle point problem, convergence, spectral analysis
2000 MSC: 65F10, 65F15, 65F50

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1. Introduction

Consider the generalized saddle point problems:

$$\mathscr{A}_+ u \equiv \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \equiv c_+, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ is a positive definite matrix, i.e., the symmetric part of A is positive definite, $B \in \mathbb{R}^{m\times n}$ is a matrix of full row rank with $m < n$, $C \in \mathbb{R}^{m\times m}$ is a symmetric positive semidefinite matrix, and $f \in \mathbb{R}^n$, $g \in \mathbb{R}^m$ are given vectors. Such systems arise from many application problems; see [1] for an overview discussions about general backgrounds and commonly used solution methods. Negativing the second block row of (1.1), it is reformulated to the equivalent variant:

$$\mathscr{A} u \equiv \begin{pmatrix} A & B^T \\ -B & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ -g \end{pmatrix} \equiv c. \tag{1.2}$$

It is indicated in [2] that the nonsymmetric formulation (1.2) is especially natural when A is nonsymmetric. In this case, the nonsymmetric matrix $\mathscr{A}$ has certain desirable properties. Especially, $\mathscr{A}$ is positive stable, i.e., the real part of all its eigenvalues are positive, see [2] for more details. The positive stable property of $\mathscr{A}$ are of great advantage when Krylov subspace methods, such as GMRES [3], are used to solve (1.2); see [4, 5].

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