Accepted Manuscript

Error analysis of the Wiener-Askey polynomial chaos with hyperbolic cross approximation and its application to differential equations with random input

Xue Luo

PII: S0377-0427(17)30627-1
DOI: https://doi.org/10.1016/j.cam.2017.12.007
Reference: CAM 11424

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 27 April 2016
Revised date : 25 September 2017

Error analysis of the Wiener-Askey polynomial chaos with hyperbolic cross approximation and its application to differential equations with random input

Xue Luo

School of Mathematics and Systems Science, Beihang University, Beijing, P. R. China, 100191 (xluo@buaa.edu.cn)

Abstract

s It is well-known that sparse grid algorithm has been widely accepted as an efficient tool to overcome the "curse of dimensionality" in some degree. In this note, we give the error estimate of hyperbolic cross (HC) approximations with all sorts of Askey polynomials. These polynomials are useful in generalized polynomial chaos (gPC) in the field of uncertainty quantification. The exponential convergences in both regular and optimized HC approximations have been shown under the condition that the random variable depends on the random inputs smoothly in some degree. Moreover, we apply gPC to numerically solve the ordinary differential equations with slightly higher dimensional random inputs. Both regular and optimized HC have been investigated with Laguerre-chaos, Charlier-chaos and Hermite-chaos in the numerical experiment. The discussion of the connection between the standard ANOVA approximation and Galerkin approximation is in the appendix.

Keywords: generalized polynomial chaos, hyperbolic cross approximation, differential equations with random inputs, spectral method
2000 MSC: 65M70

1. Introduction

Uncertainty is ubiquitous. It is usually related to the lack of knowledge about the processes involved. Although this kind of uncertainty can be reduced by obtaining more observations or by improving the accuracy of the measurements, it is quite impractical to measure at all the points, or even at a relatively large number of points. Mathematically, one usually models the uncertainty by random variables or processes, with a realistic probability distribution. The main goal in the field of uncertainty quantification is to predict the quantities of physical interest by mathematical and computational analysis. Usually, the quantity of physical interests are the real-valued functionals of the solution to certain partial/ordinary differential equations with random inputs. Generally speaking, the random inputs in the system can be expanded by an infinite combinations of random variables, say the Karhunen-Loeve expansion [10, 11] or generalized polynomial chaos (gPC). In particular, the gPC is one of the most popular approximation in the literature. The name *polynomial chaos* (PC) is coined by N. Wiener [21] in 1938, in which he studied the decomposition of Gaussian stochastic processes. The convergence of the Hermite-chaos expansion of arbitrary random processes with finite second-order moments has been shown rigorously by Cameron and Martin [5]. The study of the original PC was started by Ghanem and his coworkers. He represented the random processes by the Hermite polynomials and used this technique with finite element method to many different practical problems, see [7]. Although the Hermite-chaos is mathematically sound, the convergence rate of non-Gaussian problems are far from optimal. It is Xiu and Karniadakis [24] who for the first time generalized the Hermite polynomials to the Wiener-Askey polynomials, and numerical experiments verified the optimal convergence by choosing proper polynomial basis according to the distribution. This is so-called gPC in the literature. Later, the gPC has been further generalized to other set of complete basis, for instance the piecewise polynomial basis [2], the wavelet basis [14], and multi-element gPC [20].