Accepted Manuscript

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PII: S0377-0427(17)30637-4
DOI: https://doi.org/10.1016/j.cam.2017.12.017
Reference: CAM 11434

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 30 September 2017
Revised date : 16 December 2017

A fourth order product integration rule by using the generalized Euler-Maclaurin summation formula

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Abstract

The present paper deals with a variant of the generalized Euler-Maclaurin summation formula for the product integration rule. The main idea lies on introducing a mesh associated with the integral of the square root of the weight function. We construct the set of nodes and implement it for approximating the considered integrals. It is shown theoretically that the proposed quadrature rule is of fourth order. The results of the provided numerical examples confirm the theoretical prediction. Moreover, applications of the suggested scheme for approximating some real test problems such as weakly singular integrals, including a particular case of the Fermi-Dirac integral, are investigated.

Keywords: Generalized Euler-Maclaurin summation formula, Product integration rule, Rate of convergence, Singular integral, Fermi-Dirac integral
2010 MSC: 41A55, 65B15, 32A55, 42B20, 65D30.

1. Introduction

Mathematical models including differential and integral equations are robust and powerfull tools that have considerable importance in different branches of science and engineering. For instance, in 1823, the famous mathematician Niels Henrik Abel derived a weakly singular Volterra integral equation of the first kind while investigating the motion of a particle on a smooth curve lying on a vertical plane [9, 18]. On the other hand, the subject of fractional differential equations has received the attention of many researchers in recent years, because these equations can model the real world events more accurately according to some new features such as globality of the fractional differential and integral operators [21]. Moreover, hyper and super singular integral equations that arised from boundary integral equations analysis of engineering problems are under investigation by researchers in recent years because of their valuable importance. These singular equations are rarely harder to handle with respect to other ones and they need to some adaptive numerical schemes [13, 26, 27]. Therefore, development and improvement of the numerical methods for solving differential and integral equations should be explored to achieve the desired accuracy in computations.

Among the numerical methods for solving integral and integro-differential equations, one can refer to the quadrature methods that have essential role in computational mathematics [5, 6]. Approximating the existing integrals in both of these types of equations may be essential in any proposed numerical technique. Trapezoidal quadrature rule (or the modified trapezoidal quadrature rule) is a well-known and frequently implemented in the literature. For instance, application of modified trapezoidal quadrature scheme in the numerical solution of linear integral equations of the second kind can be found in [19]. Also, adaptive trapezoidal quadrature approach is considered for approximating some singular integrals in [15]. Other common quadrature rules such as composite midpoint [28] or in general Newton-Cotes [14] quadratures are applied for approximating the finite-part integrals in recent years.

In most of the special case studies in singular integrals, usually there exists a product of two functions as the integrand. So, implementation of appropriate quadratures for these case studies should be considered in computations. In this direction, product integration rules can be good candidates. These methods have been widely used in the literature. De Hoog et al [10] proposed a generalized Euler-Maclaurin summation formula based on piecewise Lagrangian interpolation. Also, Ma et al [17] and Alpert [2] extended high-order schemes of product integration quadratures for