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BOUNDARY CONDITIONS FOR FRACTIONAL DIFFUSION

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Abstract. This paper derives physically meaningful boundary conditions for fractional diffusion equations, using a mass balance approach. Numerical solutions are presented, and theoretical properties are reviewed, including well-posedness and steady state solutions. Absorbing and reflecting boundary conditions are considered, and illustrated through several examples. Reflecting boundary conditions involve fractional derivatives. The Caputo fractional derivative is shown to be unsuitable for modeling fractional diffusion, since the resulting boundary value problem is not positivity preserving.

1. Introduction

The space-fractional diffusion equation replaces the second derivative or Laplacian in the traditional diffusion equation with a fractional derivative. Fractional derivatives were invented soon after their integer-order counterparts, and by now have become an established field of study with a wide variety of applications in science and technology [26, 34, 45, 44, 53, 54, 61, 65]. Practical applications include physics [27, 41, 60, 72], finance [23, 32, 46, 66, 67], microbiology [6, 20, 29], medical imaging [8, 42, 43, 58, 73, 84], and hydrology [10, 15, 33, 9]. Zaslavsky [81, 82] initiated the application of fractional calculus to chaotic dynamical systems. These methods may also be applicable to nonlinear dynamics in Hodgkin-Huxley neurons and pancreatic beta cells [24, 25, 69]. Many effective numerical methods have been developed for fractional differential equations, along with proofs of stability and consistency [7, 14, 16, 22, 39, 40, 47, 48, 49, 56, 61, 62, 71, 76, 80, 83]. Because fractional

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Key words and phrases. Fractional calculus; boundary value problem; numerical solution; well-posed.

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