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Domain decomposition technique for a model of an elastic body reinforced by a Timoshenko's beam

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ABSTRACT

A body made of fiber-reinforced elastic material is considered. It is assumed that a behavior of a fiber is described by the Timoshenko beam. We propose a method of numerical solving to an equilibrium problem of the fiber-reinforced body. The method is based on the Uzawa algorithm and the domain decomposition technique. A numerical examination is carried out to demonstrate the efficiency of the proposed method.

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1. Introduction

The progress in the development of many modern engineering industries, aerospace engineering, civil engineering, and other specialties is caused by using composite materials. This is due to the fact that the composite materials increase the strength properties and carrying capacities of constructions elements while the weight of products decreases. A composite material is inhomogeneous continuous medium consisting of a few materials and bodies, the properties and behavior of which can be substantially different from each other (see, e.g., [1]). We also note that currently composite materials containing the carbon nanotubes are widely used because they have high strength and rigidity (see, e.g., [2,3]). Therefore, adequate mathematical and numerical modeling of structures made of composite materials is an actual problem. Such modeling allows to predict the behavior of composite bodies, accurately describe the characteristics, take into account nonlinear effects on fibers, formulate criteria of the occurrence and propagation of cracks.

In this paper, we consider a boundary value problem describing an equilibrium of an elastic body with a thin elastic inclusion (fiber). We use a model of a fiber-reinforced composite material proposed in [4]. It is supposed that the inclusion is modeled by a Timoshenko beam (see, e.g., [5–7]). It is located inside the body and bonded into elastic matrix. On one part of the external boundary the body is fixed, on the rest part the surface loadings act. The equilibrium problem is formulated as a minimization problem of the energy functional over the set of kinematically admissible displacements.

The main goal of the paper is to construct and test a numerical algorithm for solving the problem. The problem considered in the paper is in fact a problem of coupling of different models (model of an elastic body and model of a Timoshenko beam), in order to construct a numerical algorithm naturally to use the domain decomposition method. At each step of the iterative algorithm two problems are solved: an equilibrium problem of the elastic body without inclusions and an equilibrium problem of the Timoshenko beam. The solutions of such problems are "connected" with each other by Lagrange multipliers. Various numerical examples illustrating the feasibility of the algorithm are presented.

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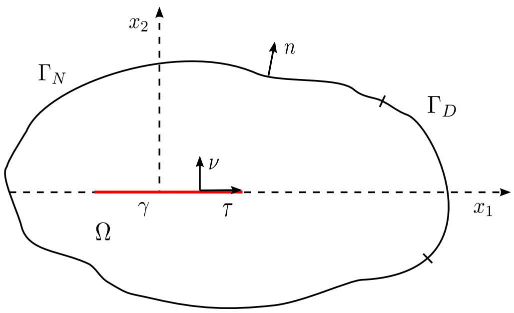

Fig. 1. Domain configuration.

Let us give a brief overview of works devoting investigation and modeling of fiber-reinforced materials. In papers [8–18], various mathematical models of elastic bodies containing thin elastic, rigid, and semi-rigid inclusions (fibers) are presented. In these works, the correctness of the corresponding boundary-value problems, the asymptotic properties of solutions, optimal control problems, and homogenization problems were investigated.

Fracture and delamination processes of fiber-reinforced composites were modeled in [19–22]. In [23], a model of non-linearly elastic solids reinforced by continuously distributed embedded fibers was investigated in which elastic resistance of the fibers to extension, bending and twist is taken into account explicitly. A problem of a Timoshenko beam of finite length loaded by concentrated forces and perfectly bonded to a homogeneous elastic and isotropic half plane was considered in [24]. An asymptotic analysis of the stress field at the beam ends was performed.

In paper [25], a constitutive model to simulate structural components with strain hardening cement-based composite under different types of loading conditions was developed. It was shown that the proposed model is mesh size independent.

In paper [26], a general solution for the static analysis of plates stiffened by arbitrarily placed parallel beams of arbitrary doubly symmetric cross section subjected to arbitrary loading taking into account shear deformation effect in both the plate and the beams was presented. An iterative numerical methods for solving this problem was constructed.

In conclusion, we note works [27–32] in which various problems of mechanics were solved numerically by using the domain decomposition method. In [33–36], the domain decomposition method was applied to problems of the crack theory. The main goal of the last mentioned papers was adapting the domain decomposition method for problems of the linear elasticity describing the equilibrium of bodies with cracks with unilateral conditions (nonpenetration conditions) imposed on the crack faces. In contrast of the present work, problems of coupling of different models have not been considered.

2. Statement of the problem

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with a Lipschitz boundary $\partial\Omega$ such that $\partial\Omega = \overline{\Gamma}_N \cup \overline{\Gamma}_D$, $\Gamma_N \cap \Gamma_D = \emptyset$ $meas\ \Gamma_D > 0$. Let $\gamma = (-0.5, 0.5) \subset \Omega$ be an interval lying on Ox_1-axis such that $\overline{\gamma} \subset \Omega$. Denote by $\boldsymbol{\nu} = (0, 1)$ and $\mathbf{n} = (n_1, n_2)$ a unit normal vector to γ and a unit normal vector to $\partial\Omega$, respectively; denote by $\boldsymbol{\tau} = (1, 0)$ a tangent vector on γ (see Fig. 1) .

In what follows, the domain Ω represents a region filled with an elastic material, and γ is an elastic inclusion incorporated in the elastic body. We assume that the behavior of the inclusion γ is described within the framework of Timoshenko's beam theory.

Denote by $\mathbf{u} = (u_1, u_2)$ a vector of displacements of the elastic body; $\sigma(\mathbf{u}) = \{\sigma_{ij}(\mathbf{u})\}$ and $\varepsilon(\mathbf{u}) = \{\varepsilon_{ij}(\mathbf{u})\}$ are a stress tensor and a strain tensor, respectively,

$$\varepsilon_{ij}(\mathbf{u}) = 1/2(u_{i,j} + u_{j,i}), \quad i, j = 1, 2.$$

Lower indices after comma denote the operation of differentiation with respect to corresponding coordinate; the summation over repeated indices is performed from 1 to 2.

Let $[h] = h|_{\gamma^+} - h|_{\gamma^-}$ be a jump of a function h on γ, where $h|_{\gamma^\pm}$ is a trace of h on $\gamma^\pm$. The signs $\pm$ fit to positive and negative directions of ν, respectively. Let w and v be horizontal (along Ox_1-axis) and vertical (along Ox_2-axis) displacements of the thin inclusion γ; let φ be a rotation angle of γ.

By $C = \{c_{ijkl}\}$, $i, j, k, l = 1, 2$, we denote a given elasticity tensor with the usual properties of symmetry and positive definiteness,

$$c_{ijkl} = c_{jikl} = c_{klij}, \quad i, j, k, l = 1, 2, \quad c_{ijkl} \in L_\infty(\Omega),$$

$$c_{ijkl}\xi_{ij}\xi_{kl} \geq c_0\|\xi\|^2 \quad \forall \xi_{ij} = \xi_{ji}, \quad c_0 = const > 0.$$

The stress tensor $\sigma(\mathbf{u})$ is defined by

$$\sigma(\mathbf{u}) = C\varepsilon(\mathbf{u}), \ i.e., \ \sigma_{ij}(\mathbf{u}) = c_{ijkl}\varepsilon_{kl}(\mathbf{u}), \quad i, j = 1, 2.$$

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