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An interior point method for nonlinear optimization with a quasi-tangential subproblem

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ABSTRACT

We present an interior point method for nonlinear programming in this paper. This method follows Byrd and Omojokun's idea of step decomposition, which splits the trial step into a normal step and a tangential step. The method employs a new idea of quasi-tangential subproblem, which is used to generate a tangential step that does not lie strictly on the tangent space of the constraints. Quasi-tangential subproblem is finally formulated into an unconstrained quadratic problem by penalizing the constraints. This method is different and maybe simpler than similar ideas, for example, the relaxed tangential step in trust funnel methods (Gould and Toint, 2010; Curtis, et al., 2017). Also, our method does not need to compute a base of the null space. A line search trust-funnel-like strategy is used to globalize the algorithm. Global convergence theorem is presented and applications to mathematical programs with equilibrium constraints are given.

1. Introduction

In this paper, we describe and analyze an interior point method for nonlinear constrained optimization problem

$$\begin{aligned} &\min f(x) \\ &\text{s.t. } c(x) = 0, \\ &\quad x \geq 0, \end{aligned} \tag{1}$$

where $f : R^n \to R$, $c : R^n \to R^m$ are smooth functions. Clearly, problems with general nonlinear inequality constraints can be equivalently reformulated into this form by using slack variables.

Interior point methods are efficient in treating inequality constraints. They have been intensively studied in the last three decades; see [1–5]. The classical interior point strategy obtains a solution by approximately solving a series of barrier problems of the form

$$\begin{aligned} &\min \varphi^{\mu}(x) \overset{\text{def}}{=} f(x) - \mu \sum_{i=1}^{n} \ln x^{(i)} \\ &\text{s.t. } c(x) = 0, \end{aligned} \tag{2}$$

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where μ is a barrier parameter which is decreasing and converges to 0. For this solution, some algorithms use Newton (or quasi-Newton) methods [6,7] while some involve SQP (or trust-region) mechanisms. The Newton-like methods get search direction from solving a Newton equation of the perturbed optimality system. Plenty of researches on methods following this algorithmic philosophy have proven their robustness and efficiency [6–9]. Also, some SQP or trust-region based interior point methods have been proposed, many of which have provided promising numerical results [2,4,10–12].

Of all the methods, the step-decomposition approaches, which integrate ideas of interior point methods and Byrd–Omojokun's trust-region idea [13,14], have got plenty of attentions because of the always consistent subproblems and the capacity of infeasibility detection [12]. Our approach follows this framework with a major character that it employs a quasi-tangential subproblem which generates a step not strictly lying on the null space of ∇c_k^T. The key point of the quasi-tangential subproblem is to convert the tangential subproblem into an unconstrained quadratic programming by penalizing the null space constraints. A quasi-tangential step satisfying some necessary conditions is obtained if choosing sufficiently small penalty factor. This strategy also circumvents the cost of computing a base for the null space, which is important in solving tangential subproblem in Byrd–Omojokun-like algorithms.

The idea of quasi-tangential subproblem has some similarity with some methods. Some methods with inexact step computation, in interior points, trust-region or SQP scheme, adopt similar ideas. Curtis et al. [4] used an inexact Newton technique in their interior point method where an inexact tangential step is generated from an inexact Newton equation for the tangential subproblem. Heinkenschloss and Ridzal [15] obtained inexact tangential steps by computing approximate projections of vectors onto the tangent space of the linearized constraints.

A more similar idea is the relaxed tangential step, which was first introduced by Gould and Toint in [16]. In [16], the authors specified conditions that a relaxed tangential step should satisfy. Recently, Curtis, Gould, Robinson and Toint [17] generalized this concept and defined the concepts of relaxed SQP tangential step and very relaxed SQP tangential step in the context of interior point trust funnel algorithm for nonlinear optimization. To achieve the very relaxed (or very relaxed) tangential step, a complex trust-region strategy is used to control the size of normal and tangential steps. The main difference between our method and the relaxed tangential step method is that our method controls the degree of violation of null space constraints directly, while relaxed tangential step controls the size of tangential step by a trust-region.

Another character of our algorithm is that we use a trust-funnel-like strategy to balance the improvements on feasibility and optimality. Trust funnel method was introduced by Gould and Toint in [16] and extended by Curtis et al. [17]. And similar ideas can be found in [18–22] etc. Our ideas mainly differ from these algorithms in the way of computing trial steps and the switch conditions between so called f-iteration and h-iteration. And the mechanism of our method seems simpler than these methods.

The balance of this paper is organized as follows. In the next section, we describe the design of the algorithm in detail. In Section 3, we show that the proposed algorithm is well-defined while the global convergence is shown in Section 4. In Section 5, we report preliminary numerical results. Finally, some further remark is given in Section 6.

Notations: We use $\|\cdot\|$ to denote the Euclidean norm $\|\cdot\|_2$. Subscript k refers to iteration indices and superscript (i) is the ith component of a vector.

2. Algorithm description

We motivate the main algorithm in this section. We first introduce the main framework of primal–dual interior point methods, then deduce the method with quasi-tangential subproblem from some ideas of trust-region methods.

2.1. The primal–dual barrier method

The Karush–Kuhn–Tucker (KKT) conditions of the barrier problem (2) cause the following nonlinear system

$$\begin{pmatrix} \nabla f(x) + \nabla c(x)\lambda - z \\ -\mu X^{-1}e + z \\ c(x) \end{pmatrix} = 0 \tag{3}$$

where $\lambda \in R^m$ and $0 \le z \in R^n$ are Lagrangian multipliers and $X = \mathrm{diag}(x_1, x_2, \ldots, x_n)$. Multiplying the second row of (3) by X, we obtain the system

$$\begin{pmatrix} \nabla f(x) + \nabla c(x)\lambda - z \\ Xz - \mu e \\ c(x) \end{pmatrix} = 0. \tag{4}$$

This may be viewed as a perturbed KKT system for the original problem (1). The optimality error for the barrier problem is defined, based on (4) as [9]

$$E_\mu(x, \lambda, z); = \left\{ \frac{\|\nabla f(x) + \nabla c(x)\lambda - z\|}{s_d}, \frac{\|Xz - \mu e\|}{s_c}, \|c(x)\| \right\}$$

with scaling parameters $s_d, s_c \ge 1$ defined as

$$s_d = \max\left\{ s_{\max}, \frac{\|\lambda\|_1 + \|z\|_1}{m+n} \right\} / s_{\max}, \quad s_c = \max\left\{ s_{\max}, \frac{\|z\|_1}{n} \right\} / s_{\max},$$