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A Weak Galerkin Finite Element Method for a Coupled Stokes-Darcy Problem on General Meshes ☆

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Abstract

In this paper, we introduce and analyze a weak Galerkin finite element method for numerically solving the coupling of fluid flow with porous media flow. Flows are governed by the Stokes equations in primal velocity-pressure formulation and Darcy equation in the second order primary formulation, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. By using the weak Galerkin approach, we consider the two-dimensional problem with the usual polynomials of degree $k \geq 1$ for the velocity and hydraulic head, while polynomials of degree $k-1$ for the pressure, the velocity and hydraulic head is enhanced by polynomials of degree $k-1$ on the edge of a finite element partition. This new method has a lot of attractive computational features: more general finite element partitions of arbitrary polygons with certain shape regularity, fewer degrees of freedom and parameter free. Stability and error estimates of optimal order are obtained. Moreover, numerical experiences are presented to illustrate the good performance, confirm the optimal order of convergence and verify the efficiency of the proposed weak Galerkin method in this paper. We also deal with finite element partitions consisting of general meshes, such as triangular mesh, quadrilateral mesh, hexagonal-dominant mesh and voronoi mesh for the numerical weak Galerkin approximation.

Keywords: Stokes-Darcy; Weak Galerkin method; Beavers-Joseph-Saffman condition; Error estimates; General meshes

1. Introduction

Numerical methods for modelling of underground flows have received increasing attention in recent years, especially the methods allowing for arbitrary-order discretizations and general meshes, due to the fact that there are degenerate elements and nonconforming interfaces account for complex geometric features resulting from compaction, erosion, and the onset of fractures or fault [1]. We could see the seminal work of the Mimetic Finite Differences [2], the Hybridizable Discontinuous Galerkin (HDG) [3, 4, 5], the Weak Galerkin (WG) Methods [6, 7], and the Hybrid High Order methods [8], and the Virtual Element Methods [9].

The WG method developed in [6, 7] for second-order elliptic problems, which is a combination of standard finite element and the idea of discontinuity by introducing the value of function on the interior and interface of elements. According to formula of integration by parts, the WG method substitutes the classical operates (e.g., gradient, divergence, curl, etc.) by the weakly defined operates. The idea of WG method has been introduced and analyzed in [6] for second-order elliptic problems based on local RT_k or BDM_k elements, numerical implementations of the WG method were discussed in [10] for some model problems with more general finite element partitions. In [11], the authors stated the WG method for the Stokes equations and proved the L^2 optimal order error estimates for pressure and the H^1 optimal order error estimates for velocity. Due to the use of the RT_k or BDM_k elements, the weak Galerkin finite element formulation was limited to classical finite element partitions of triangles or tetrahedra. Then, in [7], the WG method was extended to allow arbitrary shapes of finite elements in a partition by applying a stabilization idea, which provides a convenient flexibility in mesh generation. We refer to several papers for applications of the WG method to some other

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