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Pricing real estate index options by compactly supported radial-polynomial basis point interpolation

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ABSTRACT

This paper presents a novel method to price the real estate index options, which are modeled based on the framework proposed by Fabozzi et al. (2012). The CS–PBF method combines compactly supported radial basis functions (CSRBF) and polynomial basis functions (PBF) to yield the interpolation functions, which can guarantee interpolation shape functions with Kronecker property and overcome possible singularity associated with the PBF method. Compared with the CSRBF method and the finite difference (FD) method, the CS–PBF method is more accurate and efficient for the real estate index option. Meanwhile, a local mesh refinement technique is employed for dealing with the non-smooth options' payoffs, which is very effective and stable to improve the computational accuracy for the CS–PBF method. Finally, the CS–PBF method is extended to price American option of the real estate index.

1. Introduction

The subprime mortgage crisis in 2008 has emphasized the importance of the real estate market to the global economy, which has been labeled as a system crisis, liquidity crisis, and a crisis of confidence (in the financial markets) among others. Andersen et al. (2012) [1] surveyed how operational risk exposure in the organizations of mortgage brokers and banks, insurance companies, credit rating agencies, and investment banks contributed to the financial crisis. Their findings suggest that operational risk exposure played a crucial role in triggering the financial crisis. Therefore there was a need for hedging this risk in the real estate market. Yet, by comparison with equity, commodity, and debt markets, there has been little innovation with respect to viable derivatives products for the real estate market. Hinkelmann and Swidler (2008) [2] and Schorno et al. (2014) [3] examine whether Chicago Mercantile Exchange (CME) futures contracts can effectively be used to hedge residential real estate price risk. Since there is little evidence of any simple systematic relation between house prices and futures prices, neither static nor dynamic strategies would have maintained an effective hedge during the significant decline in housing prices. A further implication is that the Chicago Mercantile's introduction of a financial asset whose value reflects house prices will help complete the market and manage the risk in the real estate markets.

The use of Index-based futures and options for risk management in real estate markets has been discussed by Case et al. (1993) [4], Buttimer et al. (1997) [5] and Fisher (2005) [6]. Fisher [6] introduced several derivatives based on the NCREIF Property Index (NPI) and gave strategies that use these derivatives to hedge real estate risk. There are some researches for pricing the Index-based derivatives. Case et al. [4] was the first to consider perpetual futures on housing indices. Buttimer et al. [5] analyzed a two-state model for pricing derivatives contingent on a real estate index and an interest rate. Otaka and Kawaguchi (2002) [7] considered that a real estate security cannot be replicated nor hedged perfectly, so they developed a

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risk-minimising strategy to hedge and price the real estate securities under market incompleteness and obtained a reference price. Bragt et al. (2015) [8] developed a risk-neutral valuation procedure for real estate derivatives contingent on auto-correlated indices. This approach also leads to analytical pricing formulas for various real estate derivatives. An equilibrium model to price housing index derivatives has been recently developed by Geltner and Fisher (2007) [9], Cao and Wei (2010) [10]. Fabozzi et al. (2012) [11] offers a flexible and robust framework for pricing the main property derivatives contracts, which is different from the equilibrium model developed recently by Cao and Wei [10]. Their strategy is to determine the market price of risk using their model and a derivative market such as futures or forwards, which will then allow one to fix the risk-neutral pricing measure for any other derivatives such as options. Within this framework, once the market is completed with information from one property derivatives market, such as futures or forwards, all the other property derivatives contingent on the same index can be priced directly. Then they provided the analytical formula of pricing the Index-based European options. For American options or other exotic options, the closed-form solutions are not available. So, we derive the partial-differential equation (PDE) governing the real estate index derivatives price and solve it using numerical method.

Options are priced using mathematical models that are often challenging to solve. The numerical approximations for American options are needed. To this aim, the most common approaches are the binomial/trinomial trees (see, e.g., [12–14]), the finite difference (see, e.g., [15–17]), finite element and finite volume methods (see, e.g., [18–21]). Nevertheless some authors have also proposed the use of meshless algorithms based on radial basis functions [22]. This method is a truly meshless computational method, which does not require the generation of a regular grid as in the finite difference or a mesh as in the finite element methods, and the spectral accuracy is yielded (see [23]). Since Meshless methods are very easy to implement and can be applied with any node distribution, Meshless methods are a very powerful tool for solving partial differential equations.

One of the most common meshless approaches is the so-called point interpolation method (PIM), which is often based on two different types of function: Polynomial basis function (PBF) and radial basis function (RBF). The PBF method is one of the earliest interpolation schemes with the so-called Kronecker property, which allows one to easily impose essential boundary and initial (or final) conditions. But the resulting system of linear equations possibly causes singularity only with polynomial basis [24]. The RBF methods commonly used Multiquadric (MQ) or Gaussian basis functions, which are global radial basis functions. These two radial basis functions all include shape parameters. In particular, if an inappropriate shape parameter is employed, the accuracy of the result could be awful [25,26]. Moreover, the global basis function methods could yield the ill-conditioned matrix with a very large number of interpolation nodes. Wu (1995) [27] used the compactly supported radial basis functions (CSRBF) instead of the global radial basis functions, which could reduce the original resultant full matrix to a sparse matrix. The operation of the banded matrix system reduces the ill-conditioning of the resultant coefficient matrix [28]. In this manuscript, a radial-polynomial basis point interpolation is developed, which is a basis of both radial basis functions and polynomial basis functions. It overcomes possible singularity associated with the meshless methods based on only the polynomial basis. Furthermore, the obtained interpolation functions pass through all scattered points in an influence domain and retain the Kronecker property. This makes the implementation of essential boundary conditions much easier than the meshless methods based on the moving least-squares approximation. In addition, the partial derivatives of shape functions are easily obtained, which improve computational efficiency.

We remark that the main contribution of this manuscript is that the real estate index option is modeled under the framework proposed by Fabozzi et al. [11] and the PDE of the real estate index option is derived. To the best of our knowledge, the radial-polynomial basis point interpolation has rarely been used in mathematical finance. Therefore, it appears to be interesting to extend such a numerical technique to pricing real estate option, which is done in the present manuscript. Meanwhile, we can also extend the results of Fabozzi et al. [11] to American option. Second, we employ a basis of both compactly supported radial basis functions and polynomial basis functions (namely, CS–PBF) to yield the interpolation functions. CS–PBF interpolation guarantees interpolation shape functions with Kronecker property and overcomes possible singularity associated with the PBF method, then improves the accuracy for pricing the option. In particular, in order to show the efficiency and accuracy for the CS–PBF method, we test and compare the results with CSRBF and the finite difference (FD). Furthermore, in the present manuscript we also use a local mesh refinement algorithm since the options' payoffs are discontinuous at the strike price, find that the local mesh refinement effectively reduce the results' errors.

The reminder of the paper is organized as follows. In Section 2 we derive the PDE for the real estate index option. Section 3 is devoted to presenting the CS–PBF method and its application to solve the PDE. Some numerical examples based on the FD, CSRBF and CS–PBF methods and the discussion of our results are shown in Section 4. The final section summarizes our conclusions.

2. The real estate index option model

According to the empirical evidence, the property indices – commercial and residential, US or UK, appraisal or transaction based – have some characteristics: (1) there is a degree of serial correlation that induces a degree of predictability with positive autocorrelation in the short term; (2) there is evidence of negative autocorrelation for long horizons; and (3) returns of real estate portfolios are mean reverting with risk decreasing over the long horizon. So Fabozzi et al. [11] supposed that