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A WEAK GALERKIN FINITE ELEMENT METHOD FOR THE NAVIER-STOKES EQUATIONS*

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Abstract. In this paper, we propose and analyze a weak Galerkin finite element method for the Navier-Stokes equations. The new formulation hinges upon the introduction of weak gradient, weak divergence and weak trilinear operators. Moreover, by choosing the matching finite element triples, this new method not only obtains stability and optimal error estimates but also has a lot of attractive computational features: general finite element partitions of arbitrary polygons or polyhedra with certain shape regularity and parameter free. Finally, several numerical experiments assess the convergence properties of the new method and show its computational advantages.

Mathematics Subject Classification. 65N30, 65N12, 65N15, 76D05.

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1. Introduction

The Navier-Stokes equations characterize a variety of flows, including pipe flow, flow around airfoils, weather, blood flow and convective heat transfer inside industrial furnaces, which play an important role in many engineering applications. Thus, there exist many finite element approximations of the steady incompressible Navier-Stokes problems. The readers can refer to [7, 29, 33–35] and the references therein for a good description and study of a wide collection of schemes. Of course, there are the nonconforming finite element schemes [30–32], but their velocities must be continuous at the k Gauss points on triangle sides. More concretely, we list the commonly used finite element methods as follows. Continuous Galerkin FEMs [25] generally have relatively less unknowns by assuming continuous (polynomial) approximations, but they are not flexible for certain problems that have solutions with low regularity; Discontinuous Galerkin FEMs [26] can handle complicated domain and problems with low regularity by utilizing discontinuous basis functions, but they need choose appropriate penalty factors; Mixed FEMs [27, 28] solve the primal variable and flux simultaneously, but they need satisfy the inf-sup condition and require special solvers for saddle-point problems.

For the discretization of the Navier-Stokes equations, we use the weak Galerkin finite element method. The weak Galerkin method was recently introduced in [1] for second-order elliptic problems based on local *RT* or

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