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Spectral properties of the matrix splitting preconditioners for generalized saddle point problems ☆

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Abstract

In this paper, we study the spectral properties of two different kinds of preconditioners for generalized saddle point problems. One is based on the two-parameter matrix splitting preconditioner for saddle point problems by Wang et al., we generalize this preconditioner to generalized saddle point problems and analyze the spectral properties of the corresponding preconditioned matrix. The other is based on the Hermitian and skew-Hermitian splitting (HSS) preconditioner for generalized saddle point problems by Huang et al., we study the spectral properties of the HSS preconditioner with two different parameters α and β (the generalized HSS preconditioner) for generalized saddle point problems. In addition, some numerical tests are given to verify the validity of the presented theoretical results.

Keywords: Generalized saddle point problems; Preconditioner; Iteration method; GMRES

MSC: 65F10, 65F50, 65F15

1. Introduction

We consider the following linear system:

$$\mathcal{A}\mathcal{X} \equiv \begin{pmatrix} A & B^T \\ -B & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ -q \end{pmatrix} \equiv \mathcal{F}, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is nonsymmetric positive definite, i.e., $H = \frac{1}{2}(A + A^T)$ is symmetric positive definite, and matrix $B \in \mathbb{R}^{m\times n}$ has full row rank, $C \in \mathbb{R}^{m\times m}$ is symmetric positive semi-definite. $p \in \mathbb{R}^n$ and $q \in \mathbb{R}^m$ are given vectors, with $n \geq m$, B^T denotes the transpose of B. Under these conditions, the coefficient matrix $\mathcal{A}$ is nonsingular and the linear system (1) has a unique solution. This linear system (1) is called generalized saddle point problems, which have been extensively investigated for

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