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Abstract

In this paper, a new optimal cubic Hermite interpolation method is presented. The method is to optimize the derivative of the interpolant. The diagonally dominant property of the obtained system of normal equations and the error bound are better than some of the existing cubic interpolants. For parametric curve design, the vector-valued interpolation method is given. Some numerical examples are provided to illustrate the satisfactory shape of the interpolation curves.

Keywords: Hermite interpolation, fitting data, optimal property of cubic splines, derivative optimization

1. Introduction

Cubic Hermite interpolation provides an efficient and simple method for numerical approximation. Let $a = x_1 < x_2 < \cdots < x_n = b$ be a partition of an interval $[a, b]$. For $x \in [x_i, x_{i+1}], i = 1, 2, \ldots, n-1$, the cubic Hermite interpolant is

$$H(x) = (1-t)^3 y_i + 3(1-t)^2 t\left(y_i + \frac{h_i}{3} m_i\right) + 3(1-t)t^2\left(y_{i+1} - \frac{h_i}{3} m_{i+1}\right) + t^3 y_{i+1}, \tag{1}$$

where $h_i = x_{i+1} - x_i$, $t = (x - x_i)/h_i \in [0, 1]$. It is known that $H \in C^1[a, b]$ and $H(x_i) = y_i, H'(x_i) = m_i, i = 1, 2, \ldots, n$. In data-fitting problems, the values y_i are given data and the derivatives m_i remain to be determined. In order to determine the values of the derivatives m_i, let us consider the functionals

$$I_k(m_1, m_2, \ldots, m_n) = \int_a^b \left[H^{(k)}(x) - L^{(k)}(x)\right]^2 dx = \sum_{i=1}^{n-1} \int_{x_i}^{x_{i+1}} \left[H^{(k)}(x) - L_i^{(k)}(x)\right]^2 dx, \tag{2}$$

where $k = 0, 1, 2, L(x) := L_i(x)$ for $x \in [x_i, x_{i+1}]$ and $L_i(x) = (1-t)y_i + t y_{i+1}, t = (x - x_i)/h_i \in [0, 1]$.

The classical cubic spline interpolant is obtained by solving a system of the m_i so that $H \in C^2[a, b]$, see [2, 5]. For the natural cubic spline interpolant, the system is the same as the system obtained by minimizing $I_2(m_1, m_2, \ldots, m_n)$. We will give the system in the next section. It is well known that the second derivative of the natural cubic spline interpolant has the minimality property. However, the natural cubic spline interpolation curves may have unsatisfactory oscillation.

Recently, a method was presented in [3] by minimizing $I_0(m_1, m_2, \ldots, m_n)$. In the method, we don't need to choose parameters, but the obtained cubic interpolation curves may change from one side of the linear interpolation curve L to the other side. In [4], for $k = 0$, the integral interval of (2) is restricted to $x \in [x_1, x_3] \cup [x_{n-2}, x_n]$.

Different to (2), in [19], the methods of determining the derivatives m_i were presented by minimizing

$$J_k(m_1, m_2, \ldots, m_n) = \int_a^b [H^{(k)}(x)]^2 dx, \quad k = 0, 1, 2, 3.$$