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Rational General Solutions of Systems of First-Order Algebraic Partial Differential Equations

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Abstract

We study the rational solutions of systems of first-order algebraic partial differential equations and relate them to those of an associated autonomous system. We also describe how rational general solutions of these systems are related, and provide an algorithm in some particular case concerning the dimension of the associated algebraic variety. Our results can be considered as a generalization of the approach by L. X. C. Ngô and F. Winkler on algebraic ordinary differential equations of order one, adapted to systems of first-order algebraic partial differential equations.

Keywords: Algebraic partial differential equation, rational general solution, exact computation

1. Introduction

The study of solutions of systems of partial differential equations in the complex domain goes along a parallel study of that of a single equation. This is a classical approach in mathematics, which has proved successful in many circumstances. There are widespread approximation techniques of the solutions by different methods. For a historical review on the analytic approach we refer to [5]. In this paper, we study exact symbolic solutions of systems of algebraic partial differential equations. Our work is based on differential algebra techniques, put forward in [20, 25].

More precisely, our main aim is to investigate the exact rational solutions of systems of algebraic partial differential equations (APDEs, for short) from the point of view of an algebro-geometric treatment. We deal with systems of first-order partial differential equations. To such a system we associate an algebraic variety. If this associated variety admits a rational parametrization, we derive information on the rational solvability from such a parametrization.

An algebraic-geometric method for solving differential equations was proposed in [15] via Gröbner bases; see also [26]. Several advances have been made concerning not only the treatment of equations of higher order, but also the extension of results for autonomous equations to more general ones. Algebraic ordinary differential equations (AODEs, for short) were considered in [6, 7], where the authors develop an algorithm for deciding whether an autonomous first-order AODE admits a rational solution, and, in the affirmative case, for computing it. These investigations have been extended to radical solutions in [10, 12], to the non-autonomous case in [22, 24, 23], and to higher order AODEs in [14].

In the case of systems of ordinary differential equations, a first step is taken in [21] when studying systems of AODEs of algebro-geometric dimension one. A similar direction is explored in [13].

The multivariate setting of the problem deals with partial differential equations. The development concerning APDEs has progressed in a manner analogous to AODEs. In [11] we describe a solution method

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