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From Zhang Neural Network to scaled hyperpower iterations

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Abstract

A class of scaled hyperpower iterative methods for computing outer inverses is considered. This class appears during the construction of the discrete-time Zhang neural network for computing the usual matrix inverse. The usual hyperpower iterative methods belong to this class. Additionally, a more general class of scaled iterative methods, which includes the scaled hyperpower method, is defined and studied. Different values of the real scaling parameter are investigated both theoretically and numerically.

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Key words: Outer inverse, Discrete Zhang Neural Networks, Moore-Penrose inverse, iterative methods, convergence.

1 Introduction and preliminaries

Following the usual notation, the set of all $m \times n$ complex matrices of rank r is denoted by $\mathbb{C}_r^{m\times n}$. Further, $||A||_2$, A^*, $\mathcal{R}(A)$ and $\mathcal{N}(A)$ denote the matrix 2-norm, the conjugate transpose, the range space and the null space of $A \in \mathbb{C}^{m\times n}$. In addition, $\text{rank}(A)$ denotes the rank of a given matrix A. For $A\in\mathbb{C}^{n\times n}$, the smallest nonnegative integer j such that $\text{rank}(A^{j+1}) = \text{rank}(A^j)$ is called the index of A and is denoted by $\text{ind}(A)$. The key point in the investigation and computation of generalized inverses of $A\in\mathbb{C}_r^{m\times n}$ leads to, so called, Penrose equations

$$(1)\quad AXA = A \qquad (2)\quad XAX = X \qquad (3)\quad (AX)^* = AX \qquad (4)\quad (XA)^* = XA$$

for unknown matrix X. By $A\{\mathcal{S}\}_s$ we denote the set of all $\mathcal{S}$-inverses of A which are of rank s. A single element of the set $A\{1, 2, 3, 4\}_r$ is called the Moore-Penrose inverse $A^\dagger$ of A.

The Drazin inverse of a square matrix $A \in \mathbb{C}^{n\times n}$ is the unique matrix $X \in \mathbb{C}^{n\times n}$ which fulfills the matrix equation (2) in conjunction with

$$(1^k)\quad A^{l+1}X = A^l,\ l \geq \text{ind}(A), \qquad (5)\quad AX = XA,$$

and it is denoted by $X = A^D$ (for more details see [17]). In the case $\text{ind}(A) = 1$, the Drazin inverse becomes the group inverse $X = A^\#$ The outer inverse of $A \in \mathbb{C}_r^{m\times n}$ with prescribed range T and null space S, denoted by $A^{(2)}_{T,S}$, satisfies the matrix equation (2) and two additional properties: $\mathcal{R}(X) = T$

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