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Approximate solution of nonlinear quadratic integral equations of fractional order via piecewise linear functions

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Abstract

In this paper, the main objective is to describe an approximate scheme for solving nonlinear quadratic integral equations (NQIEs) of fractional order. The method is based on piecewise linear functions that are called hat functions (HFs). By applying the HFs approximation, their properties and operational matrices, nonlinear equation becomes to a nonlinear system of equations which can be solved by using simple arithmetic methods. Also, the error analysis of HFs method and convergence analysis of proposed model are given. At the end, two nonlinear examples are presented to illustrate the validity and applicability of the explained approach.
Keywords: Hat functions; Nonlinear quadratic integral equation; Fractional calculus; Operational matrix; Error analysis.

AMS Subject Classification: 45G10, 26A33, 40C05, 97N20.

1 Introduction

Fractional integrals have numerous applications in various fields, including physiology, physics, nonlinear oscillation, fluid dynamic traffic, signal processing and control theory [1-7]. In many cases, it is difficult to obtain analytical solutions of fractional integral equations. So, numerical methods as an efficient approximation method for solving fractional integral equations are of interest to many researchers [8-17].
Particularly, in recent years quadratic integral equations (QIEs) have arisen increasingly in various problems of the real world such as the theory of radiative transfer, kinetic theory of gases, the traffic theory and in the theory of neutron transport [18]. Numerical solution of these equations have been studied by many researchers. For example, Adomian solution [19], modified hat functions method [20], Adomian decomposition method (ADM) and repeated trapezoidal methods [21], ADM and numerical implementation technique (NIT) [22], Picard and Adomian methods [23], Picard and Adomian decomposition methods [24].
In this paper, we introduce HFs operational matrices to investigate the numerical solution of NQIE of fractional order as follows

$$f(x) = g(x) + \Big(\frac{1}{\Gamma(\alpha)}\int_0^x (x-y)^{\alpha-1}k_1(x,y)U_1(y,f(y))dy\Big)\Big(\frac{1}{\Gamma(\beta)}\int_0^x (x-y)^{\beta-1}k_2(x,y)U_2(y,f(y))dy\Big), \quad (1)$$

where $x \in I = [0,1]$ and $f(x) \in C^2(I)$ is an unknown function.
For convenience, we define the nonlinear terms as follows

$$U_1(x,f(x)) = [f(x)]^{p_1},$$
$$U_2(x,f(x)) = [f(x)]^{p_2},$$

where p_1 and p_2 are positive integer numbers and $U_i(x,f(x)) \in C^2(I\times\mathbb{R})$ for $i=1,2$. Moreover $g(x) \in C^2(I)$ and $k_i(x,y) \in C^2(I\times I)$ for $i=1,2$, are known functions.
For solving NQIE of fractional order, we obtain the approximate solution of $f(x)$ in truncated HFs series as

$$f_n(x) = \sum_{i=0}^{n} f_i h_i(x),$$

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