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Highly efficient iterative algorithms for solving nonlinear systems with arbitrary order of convergence $p+3$, $p \geq 5$ ☆

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Abstract

It is known that the concept of optimality is not defined for multidimensional iterative methods for solving nonlinear systems of equations. However, usually optimal fourth-order schemes (extended to the case of several variables) are employed as starting steps in order to design higher order methods for this kind of problems. In this paper, we use a non-optimal (in scalar case) iterative procedure that is specially efficient for solving nonlinear systems, as the initial steps of an eighth-order scheme that improves the computational efficiency indices of the existing methods, as far as the authors know. Moreover, the method can be modified by adding similar steps, increasing the order of convergence three times per step added.

This kind of procedures can be used for solving big-sized problems, such as those obtained by applying finite differences for approximating the solution of diffusion problem, heat conduction equations, etc. Numerical comparisons are made with the same existing methods, on standard nonlinear systems and Fisher's equation by transforming it in a nonlinear system by using finite differences. From these numerical examples, we confirm the theoretical results and show the performance of the proposed schemes.

Keywords: Nonlinear systems; iterative method; convergence; efficiency index; Fisher's equation.

1. Introduction

Nonlinearity is ubiquitous in physical phenomena as fluid and plasma mechanics, gas dynamics, elasticity, relativity, chemical reactions, combustion, ecology, biomechanics, economics modeling problems, transport theory and many other problems that are modeled by nonlinear equations. So, the design of fixed point iterative methods for solving systems of nonlinear equations is a challenging task in Numerical Analysis.

The proliferation of iterative methods for solving nonlinear equations has been spectacular in the last years (we can see a good overview in [2, 16]). Some of these methods can be transferred directly to the context of nonlinear systems, keeping the order of convergence, but others, at least apparently, cannot be extended to multidimensional case (although it can be done by using divided differences operator, as it was done in [1, 6, 8]). Other times, the procedures are designed specifically for multidimensional problems, as it is the case.

We will focus our efforts in finding the solution $\bar{x}$ of a nonlinear system $F(x) = 0$, wherein $F : D \subset \mathbb{R}^n \to \mathbb{R}^n$ is a sufficiently Fréchet differentiable function in an open convex set D. Although the most used method for finding the solution $\bar{x} \in D$ is Newton's scheme,

$$x^{(k+1)} = x^{(k)} - [F'(x^{(k)})]^{-1}F(x^{(k)}),\ k = 0, 1, 2, \ldots,$$

where $F'(x^{(k)})$ is the Jacobian matrix of function F evaluated in the kth iteration, in recent years many iterative methods have been designed for solving multidimensional nonlinear problems as [5, 7, 11, 12, 13, 17] and the references therein.

In what follows we present some recently known methods of eighth-order of convergence that will be used in the comparison with our proposed scheme, in the computational efficiency and in the numerical tests. The first one is due to

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