Choosing the most stable members of Kou's family of iterative methods

Alicia Cordero, Lucia Guasp, Juan R. Torregrosa

PII: S0377-0427(17)30070-5
DOI: http://dx.doi.org/10.1016/j.cam.2017.02.012
Reference: CAM 11014

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 22 September 2016

Choosing the most stable members of Kou's family of iterative methods ☆

Alicia Cordero[a,*], Lucia Guasp[a], Juan R. Torregrosa[a]

[a] *Instituto Universitario de Matemática Multidisciplinar*
Universitat Politècnica de València
Camino de Vera s/n, 46022 València, Spain

Abstract

In this manuscript, we analyze the dynamical anomalies of a parametric family of iterative schemes designed by Kou et al. It is known that its order of convergence is three for any arbitrary value of the parameter, but it has order four (and it is optimal in the sense of Kung-Traub's conjecture) when an specific value is selected. Among all the elements of this family, one can choose this fourth-order element or any of the infinite members of third order of convergence, if only the speed of convergence is considered. However, the stability of the methods plays an important role in their reliability when they are applied on different problems. This is the reason why we analyze in this paper the dynamical behavior on quadratic polynomials of the mentioned family. The study of fixed points and their stability, joint with the critical points and their associated parameter planes, show the richness of the class and allows us to find members of it with excellent numerical properties, as well as other ones with very unstable behavior. Some test functions are analyzed for confirming the theoretical results.

Keywords: Nonlinear equation; iterative method; dynamical behavior; Fatou and Julia sets; basin of attraction; periodic orbits.

1. Introduction

Nonlinear equations $f(z) = 0$, where $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is a real function defined in an open interval I, are often used for modeling real problems arising in science and engineering as, for example, in the analysis of dynamical models of chemical reactors [7], preliminary orbit determination of satellites [3], in radioactive transfer [13], to simulate flow transport in a pipe [24] or even the approximation of the eigenvalues of square matrices, which is known has many applications in areas as image processing, dynamical systems, control theory, etc (see, for example, [8]).

For solving these equations, iterative schemes must be used. The best known iterative approach is Newton's method. In last decades, many researchers have proposed different iterative methods to improve Newton's scheme (see, for example, the review [21], and the references therein). These variants of Newton's method have been designed by means of different techniques, providing in the most of cases multistep schemes. Some of them come from Adomian decomposition (see [1], for example). Another procedure to develop iterative methods is the replacement of the second derivative in Chebyshev-type methods by some approximation, [23]. A common way to generate new schemes is the direct composition of known methods with a later treatment to reduce the number of functional evaluations. For example, by composing Newton's method with itself, holding the derivative "frozen" in the second step, third-order Traub's method [23] is obtained.

Recently, the weight-function procedure has been used to increase the order of convergence of known methods ([21, 22]), allowing to get optimal methods, under the point of view of Kung-Traub's conjecture [17]. These authors conjectured that an iterative methods, without memory, which uses d functional evaluations per iteration can reach, at most, order of convergence 2^{d-1}. When this bound is reached, the scheme is called optimal. Although the aim of

☆This research was supported by Ministerio de Economía y Competitividad MTM2014-52016-C2-2-P and Generalitat Valenciana PROMETEO/2016/089.

*Corresponding author
Email addresses: acordero@mat.upv.es (Alicia Cordero), luguaal@ade.upv.es (Lucia Guasp), jrtorre@mat.upv.es (Juan R. Torregrosa)