ARTICLE IN PRESS

Journal of Computational and Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Adaptive morphological filters based on a multiple orientation vector field dependent on image local features

Álvar-Ginés Legaz-Aparicio [a], Rafael Verdú-Monedero [a,*], Jesús Angulo [b]

[a] Universidad Politécnica de Cartagena, Cartagena, 30202, Spain

[b] MINES ParisTech, PSL-Research University, CMM-Centre de Morphologie Mathématique, France

ARTICLE INFO

Article history:
Received 30 November 2016
Received in revised form 29 March 2017

Keywords:
Mathematical morphology
Adaptive morphology
Adaptive structuring elements
Spatially-variant structuring elements

ABSTRACT

This paper addresses the formulation of adaptive morphological filters based on spatially-variant structuring elements. The adaptivity of these filters is achieved by modifying the shape and orientation of the structuring elements according to a multiple orientation vector field. This vector field is provided by means of a bank of directional openings which can take into account the possible multiple orientations of the contours in the image. After reviewing and formalizing the definition of the spatially-variant dilation, erosion, opening and closing, the proposed structuring elements are described. These spatially-variant structuring elements are based on ellipses which vary over the image domain adapting locally their orientation according to the multiple orientation vector field and their shape (the eccentricity of the ellipses) according to the distance to relevant contours of the objects. The proposed adaptive morphological filters are used on gray-level images and are compared with spatially-invariant filters, with spatially-variant filters based on a single orientation vector field, and with adaptive morphological bilateral filters. Results show that the morphological filters based on a multiple orientation vector field are more adept at enhancing and preserving structures which contains more than one orientation.

1. Introduction

Mathematical morphology is a nonlinear image processing methodology useful for solving efficiently many image analysis tasks [1]. From a mathematical point of view, it is based on two basic operators, dilation and erosion, which correspond respectively to the convolution in the max-plus algebra and its dual convolution. More precisely, in Euclidean (translation invariant) mathematical morphology, the pair of adjoint and dual operators dilation (sup-convolution) $(f \oplus b)(x)$ and erosion (inf-convolution) $(f \ominus b)(x)$ of an image $f : E \subset \mathbb{R}^n \to \overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ are given by [2,3]:

$$\begin{cases} \delta_b(f)(x) = (f \oplus b)(x) = \sup\limits_{y \in E} \{f(y) + b(y - x)\}, \\ \varepsilon_b(f)(x) = (f \ominus b)(x) = \inf\limits_{y \in E} \{f(y) - b(y + x)\}, \end{cases} \quad (1)$$

where $b : \mathbb{R}^n \to \overline{\mathbb{R}}$ is the structuring function which determines the effect of the operator. The structuring function plays a similar role to the kernel in classical linear filtering using convolution. By allowing infinity values, the further convention for ambiguous expressions should be considered: $f(y) + b(x - y) = -\infty$ when $f(y) = -\infty$ or $b(x - y) = -\infty$, and that $f(y) - b(y + x) = +\infty$ when $f(y) = +\infty$ or $b(y + x) = -\infty$. We easily note that both are invariant under translations

* Corresponding author.
E-mail address: rafael.verdu@upct.es (R. Verdú-Monedero).

http://dx.doi.org/10.1016/j.cam.2017.05.001
0377-0427/

Please cite this article in press as: Legaz-Aparicio , et al., Adaptive morphological filters based on a multiple orientation vector field dependent on image local features, Journal of Computational and Applied Mathematics (2017), http://dx.doi.org/10.1016/j.cam.2017.05.001.

(i.e., commute with the translation operator) in the spatial ("horizontal") space E and in the image intensity ("vertical") space $\overline{\mathbb{R}}$, i.e.,

$$f(x) \mapsto f_{(y,\alpha)}(x) = f(x-y) + \alpha, \tag{2}$$

with $y \in E$ and $\alpha \in \overline{\mathbb{R}}$, then

$$\delta_b(f_{(y,\alpha)})(x) = \delta_b(f)(x-y) + \alpha. \tag{3}$$

The structuring function is typically a parametric family $b_t(x)$, where $t > 0$ is the scale parameter. In particular, the canonical structuring function is the paraboloidal shape (i.e., square of the Euclidean distance) [4,5]:

$$p_t(x) = -\frac{\|x\|^2}{2t}. \tag{4}$$

Due to its properties of semigroup, dimension separability and invariance to transform domain, the structuring function $p_t(x)$ plays a similar role to the Gaussian kernel in (linear) convolution-based filtering. The corresponding quadratic dilation and erosion by $p_t(x)$ are related to the following initial-value Hamilton–Jacobi first-order partial differential equation (Hamilton–Jacobi PDE) [6,7]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \pm\dfrac{1}{2}\|\nabla u\|^2, & x \in \mathbb{R}^n, \quad t > 0 \\ u(x,0) = f(x), & x \in \mathbb{R}^n. \end{cases} \tag{5}$$

This Hamilton–Jacobi PDE does not admit classic (i.e., everywhere differentiable) solutions but can be studied in the framework of the theory of viscosity solutions [8]. It is well known that the solutions of the Cauchy problem (5) are given by the so-called Hopf–Lax–Oleinik formulas [9], for + sign and − sign, respectively:

$$u(x,t) = \sup_{y\in\mathbb{R}^n}\left\{f(y) - \frac{\|x-y\|^2}{2t}\right\} = (f \oplus p_t)(x) \quad (\text{for } + \text{ sign}), \tag{6}$$

$$u(x,t) = \inf_{y\in\mathbb{R}^n}\left\{f(y) + \frac{\|x-y\|^2}{2t}\right\} = (f \ominus p_t)(x) \quad (\text{for } - \text{ sign}). \tag{7}$$

Such PDE model is fundamental to continuous mathematical morphology, and research on numerical schema for the solution of spatially-variant counterparts of (5) is still active [10].

The theory of morphological filtering is based on the opening $(f \circ b)(x)$ and closing $(f \bullet b)(x)$ operators, obtained respectively by the composition product of erosion–dilation and dilation–erosion using the same structuring function, i.e.,

$$\begin{cases} \gamma_b(f)(x) = (f \circ b)(x) = ((f \ominus b) \oplus b)(x) = \sup_{z\in E}\inf_{y\in E}\{f(y) - b(y-z) + b(z-x)\}, \\ \varphi_b(f)(x) = (f \bullet b)(x) = ((f \oplus b) \ominus b)(x) = \inf_{z\in E}\sup_{y\in E}\{f(y) + b(z-y) - b(x-z)\}. \end{cases} \tag{8}$$

In order to have a better insight of the effect of the opening and the closing of a function, let us rewrite $(f \circ b)(x)$ as follows:

$$\gamma_b(f) = \bigvee\left\{b_{(y,\alpha)} \mid (y,\alpha) \in E \times \overline{\mathbb{R}}, \;\; b_{(y,\alpha)} \le f\right\}, \tag{9}$$

where $\bigvee$ denotes the supremum. Therefore, in the product space $E \times \overline{\mathbb{R}}$ the subgraph of the opening is generated by the upper envelope of the horizontally and vertically translated shape function $b(x-y)+\alpha$ under the function f. In other words, function $(f \circ b)(x)$ can be seen as the supremum of the invariants parts of f under-swept by b. Regarding the closing $(f \bullet b)(x)$, a similar geometric dual interpretation is obtained:

$$\varphi_b(f) = \bigwedge\left\{\check{b}_{(y,\alpha)} \mid (y,\alpha) \in E \times \overline{\mathbb{R}}, \;\; \check{b}_{(y,\alpha)} \ge f\right\}, \tag{10}$$

where $\bigwedge$ denotes the infimum and $\check{b}(x) = -b(-x)$. This expression corresponds to the invariant parts of f over-swept by the horizontally and vertically symmetric structuring function $\check{b}$. From (9), it is straightforward to see that the opening is (i) increasing, (ii) idempotent and (iii) anti-extensive, i.e., $\forall x$, (i) $f(x) \le g(x) \Rightarrow \gamma_b(f)(x) \le \gamma_b(g)(x)$; (ii) $\gamma_b(\gamma_b(f)) = \gamma_b(f)$; and (iii) $\gamma_b(f)(x) \le f(x)$. From (10), the closing is increasing and idempotent, but being extensive: $\varphi_b(f)(x) \ge f(x)$, $\forall x$. More complex filters can be obtained by composition of openings and closings [2,3].

At this point, it could be interesting for a general reader to compare these morphological operators to the most extended family of filters based on the standard convolution of a function f by a translation-invariant kernel k:

$$(f * k)(x) = \int_E f(y)k(y-x)dy, \tag{11}$$

and in particular, to the case of canonical kernel in linear filtering, the so-called Gaussian kernel at scale t:

$$(f * g_t)(x) = C\int_E f(y)\exp\left(-\frac{\|x-y\|^2}{2t}\right)dy, \quad C = (2\pi t)^{-n/2}. \tag{12}$$