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A General Framework for a Class of Non-linear Approximations with Applications to Image Restoration

V. Candela[a,2], A. Falcó[b,1], Pantaleón D. Romero[1,1,2,*]

[a] *Departament de Matemátiques, Campus de Burjassot, Universidad de Valencia, Carrer del Dr. Moliner 50, 46100 Burjassot (Valencia), Spain*

[b] *Departamento de Departamento de Matemáticas, Física y Ciencias Tecnológicas, Universidad CEU Cardenal Herrera, Carrer San Bartolomé 55, 46115 Alfara del Patriarca (Valencia), Spain.*

Abstract

In this paper, we establish sufficient conditions for the existence of optimal non-linear approximations to a linear subspace generated by a given weakly-closed (non-convex) cone of a Hilbert space. Most non-linear problems have difficulties to implement good projection-based algorithms due to the fact that the subsets, where we would like to project the functions, do not have the necessary geometric properties to use the classical existence results (such as convexity, for instance). The theoretical results given here overcome some of these difficulties. To see this we apply them to a fractional model for image deconvolution. In particular, we reformulate and prove the convergence of a computational algorithm proposed in a previous paper by some of the authors. Finally, some examples are given.

Keywords: Non-linear approximation, Fractional Deconvolution, Image Restoration, Weakly-closed non-convex cone.

1. Introduction

Unlike linear approximation, where there exists a solid theoretical background establishing conditions for existence, uniqueness and algorithmic issues, non-linear approximation is a field with not so deep knowledge about the above related concepts. The main drawback of the non-linear approximation theory is given by the fact that most of the fundamental concepts used in the theory of linear spaces cannot be generalized without losing their strength. Lack of the vectorial structure of the spaces is in fact one of the main difficulties to obtain

*Corresponding author
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