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On Global Convergence of Gradient Descent Algorithms for Generalized Phase Retrieval Problem

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Abstract

In this paper, we study the generalized phase retrieval problem: to recover a signal $\boldsymbol{x} \in \mathbb{C}^n$ from the measurements $y_r = |\langle \boldsymbol{a}_r, \boldsymbol{x} \rangle|^2$, $r = 1, 2, \ldots, m$. The problem can be reformulated as a least-squares minimization problem. Although the cost function is nonconvex, the global convergence of gradient descent algorithms from a random initialization is studied, when m is large enough. We improve the known result of the local convergence from a spectral initialization. When the signal $\boldsymbol{x}$ is real-valued, we prove that the cost function is local convex near the solution $\{\pm \boldsymbol{x}\}$. To accelerate the gradient descent, we review and apply several efficient line search methods with exact line search stepsize. We also perform a comparative numerical study of the line search methods and the alternative projection method. Numerical simulations demonstrate the superior ability of LBFGS algorithm than other algorithms.

Keywords: Phase retrieval, Gradient descent, Global convergence, LBFGS, Local convexity
2010 MSC: 49N45, 49N30

1. Introduction

Phase retrieval is to recover a complex signal from its Fourier intensity. This problem arises in many engineering and science applications, such as X-ray crystallography [1], electron microscopy [2], X-ray diffraction imaging [3], optics [4] and astronomy [5], just to name a few. In these applications, one often has recorded the Fourier transform intensity of a complex signal, while the phase information is infeasible. Due to the absence of Fourier phase, the available information is incomplete. It has been proved that the one-dimensional phase retrieval problem suffers from essential nonuniqueness, and the multi-dimensional case is usually less prone to multiple solutions [6, 7]. However, these theories did not lead efficient recovery algorithms with convergence guarantee.

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