Journal of Computational and Applied Mathematics 314 (2017) 40–60

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Iterative coupling of flow, geomechanics and adaptive phase-field fracture including level-set crack width approaches

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HIGHLIGHTS

- Fixed-stress formulations for fluid-filled phase-field fractures.
- Accurate fracture-width finite element computations using level-sets.
- Numerical examples in 2D and 3D with spatial and temporal convergence studies.

ARTICLE INFO

Article history:
Received 18 May 2016
Received in revised form 2 September 2016

Keywords:
Fluid-filled phase field fracture
Fixed stress splitting
Pressure diffraction equation
Level-set method
Crack width
Porous media

ABSTRACT

In this work, we present numerical studies of fixed-stress iterative coupling for solving flow and geomechanics with propagating fractures in a porous medium. Specifically, fracture propagations are described by employing a phase-field approach. The extension to fixed-stress splitting to propagating phase-field fractures and systematic investigation of its properties are important enhancements to existing studies. Moreover, we provide an accurate computation of the fracture opening using level-set approaches and a subsequent finite element interpolation of the width. The latter enters as fracture permeability into the pressure diffraction problem which is crucial for fluid filled fractures. Our developments are substantiated with several numerical tests that include comparisons of computational cost for iterative coupling and nonlinear and linear iterations as well as convergence studies in space and time.

1. Introduction

Iterative coupling has received great importance for coupling flow and mechanics in subsurface modeling, environmental and petroleum engineering problems [1–7]. Recently, the extension of iterative coupling to fractured porous media has

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http://dx.doi.org/10.1016/j.cam.2016.10.022

been of interest [8–10]. However, reliable and efficient numerical methods in coupled poromechanics, including fractures, still pose computational challenges. The applications include multiscale and multiphysics phenomena such as reservoir deformation, surface subsidence, well stability, sand production, waste deposition, pore collapse, fault activation, hydraulic fracturing, CO_2 sequestration, and hydrocarbon recovery.

On the other hand, quasi-static brittle fracture propagation using variational techniques has attracted attention in recent years since the pioneering work in [11,12]. The numerical approach [11] is based on Ambrosio–Tortorelli elliptic functionals [13,14]. Here, discontinuities in the displacement field $\mathbf{u}$ across the lower-dimensional crack surface are approximated by an auxiliary function φ. This function can be viewed as an indicator function, which introduces a diffusive transition zone between the broken and the unbroken material. This zone has a half bandwidth ε, which is a model regularization parameter. From an application viewpoint, two situations are of interest for given fracture(s): first, observing the variation of the fracture width (crack opening displacement) and second, change of the fracture length. The latter situation is by far more complicated. However, both configurations are of importance and variational fracture techniques can be used for both of them.

Fracture evolutions satisfy a crack irreversibility constraint such that the resulting system can be characterized as a variational inequality. Our motivation for employing such a variational approach is that fracture nucleation, propagation, kinking, and the crack morphology are automatically included in the model. In addition, explicit remeshing or reconstruction of the crack path is not necessary. The underlying equations are based on principles arising from continuum mechanics that can be treated with (adaptive) Galerkin finite elements. An important modification of [12] towards a thermodynamically-consistent phase-field fracture model has been accomplished in [15,16]. These approaches have been extended to pressurized fractures in [17,18] that include a decoupled approach and a fully-coupled technique accompanied with rigorous analysis. Moreover, a free energy functional was established in [17]. In these last studies, the crack irreversibility constraint has been imposed through penalization. It is well-known that the energy functional of the basic displacement/phase-field model is non-convex and constitutes a crucial aspect in designing efficient and robust methods. Most approaches for coupling displacement and phase field are sequential, e.g., [19–22]; however it is well-known that a monolithic treatment has higher robustness (and potentially better efficiency) than sequential coupling. Indeed it has been shown in [23,24] that for certain phase-field fracture configurations partitioned coupling is more expensive than a monolithic solution. Consequently in this paper, we adopt a quasi-monolithic approach using an extrapolation in the phase-field variable [25,26].

Recent advances and numerical studies for treating multiphysics phase-field fracture include the following; thermal shocks and thermo-elastic–plastic solids [27–29], pressurized fractures [30,18,31–33], fluid-filled (i.e., hydraulic) fractures [9,34–39], proppant-filled fractures [40], a fractured well-model within a reservoir [41], and crack initiations with microseismic probability maps [42]. These studies demonstrate that phase-field fracture has great potential to tackle practical field problems.

Addressing multiphysics problems requires careful design of the solution algorithms. For the displacement/phase-field subproblem, we employ a quasi-monolithic approach as previously mentioned, but to couple this fractured-mechanics to flow, we use a splitting approach. The latter is more efficient for solvers and for choosing different time scales for both mechanics and flow, respectively. In addition, the splitting permits easier extensions to multiphase flow including equation of state (EOS) compositional flow. A successful splitting approach is fixed-stress iterative coupling, which has been applied in a series of papers for coupling phase-field fracture/mechanics and flow [9,34,40,42]. However a systematic investigation of the performance of this scheme is still missing. It is the objective of this paper to illustrate using benchmarks and mesh refinement studies to establish the robustness and efficiency of the fixed-stress algorithm. These studies are essential for future extensions including efficient three-dimensional practical field problems.

In addition, we also focus on a more accurate approximation of the fracture width. The authors of [43] recently proposed a two-stage level-set approach in which first a level-set function is computed with the help of the phase-field function and in a second step this level-set function is smoothed due to high gradients. However, since we only need the level-set function to obtain normal vectors on the fracture boundary, we also propose an alternative method which simplifies the above approach by avoiding the computation of an explicit level-set function but directly using the computed phase-field function. In addition, we note that these approaches do not derive the width formulation inside the fracture, which is crucial for the fracture permeability computation for fluid filled fracture propagations. Thus, here we propose a method to compute the crack width values inside the fracture by employing an interpolation based on the values in the diffusive fracture zone.

To resulting fluid filled fracture propagation framework consists of five equations (four equations when using phase-field directly as a level-set value) for five (i.e., four) unknowns: vector-valued displacements $\mathbf{u}$, scalar-valued phase-field φ, pressure p, level-set φ_{LS}, and a finite element representation of the fracture width w. The first problem (namely the displacement/phase-field) is nonlinear and subject to an inequality constraint-in-time (the crack irreversibility) whereas the other three problems are linear.

The outline of this paper is as follows: In Section 2 we recapitulate the flow equations in terms of a pressure diffraction problem, and the displacement-phase-field system for the mechanics part. In Section 3, two equations for computing a level-set function and the width are formulated. In Section 4 we discuss the discretization of all problems. In the next Section 5 we address discretization and the fixed-stress coupling algorithm. Several numerical examples are presented in Section 6, which demonstrate the performance of our algorithmic developments.