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Nonexistence of global solutions for a fractional differential problem

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is a special case of (1) with $\alpha = \beta = 1$ and $\gamma > -1$. This problem has, for $m > 1$, the solution

$$y(t) = \left[\frac{1-m}{1+\gamma} t^{1+\gamma} + b^{1-m}\right]^{1/(1-m)}.$$

Observe that, for $m > 1$, the solution blows-up in finite time.
When $\alpha = 1$, $\beta = 0$ and $\gamma = 0$, the problem (1) with an equality instead of inequality is equivalent to the Bernoulli differential problem

$$\begin{cases} y'(t) + y(t) = y^m(t), \ t > 0, \\ y(t)\,|_{t=0} = b. \end{cases} \tag{2}$$

The solution of (2) is given by

$$y(t) = \left[1 + \left(b^{1-m} - 1\right)\exp(m-1)t\right]^{1/(1-m)}.$$

Clearly $y(t)$ blows up in the finite time

$$c = \frac{1}{1-m}\ln\left(1 - b^{1-m}\right), \ m, \ b > 1.$$

In case $\alpha = \beta$, $0 < \alpha < 1$, and Riemann-Liouville fractional derivative in (1) we obtain the problem with only one fractional derivative

$$\begin{cases} 2D_0^\alpha y(t) \geq t^\gamma \left|y(t)\right|^m, \ t > 0, \\ I_0^{1-\alpha} y(t)\,|_{t=0} = b. \end{cases} \tag{3}$$

Problem (3) has been considered by Laskri and Tatar [23]. It was shown that if $\gamma > -\alpha$ and $1 < m \leq \frac{\gamma+1}{1-\alpha}$, then, Problem (3) does not admit global nontrivial solutions when $b \geq 0$.
Here, we would like to investigate the case where a lower order fractional derivative is present in the equation (or inequality). It is known that for hyperbolic equations, say the wave equation with an internal frictional damping represented by the first derivative (i.e. $\alpha = 2$, $\beta = 1$ also known as the Telegraph equation), this damping has a dissipation effect. It will compete with the polynomial source and may take it over this blowing-up term under certain circumstances. Moreover, it has been shown for the telegraph problem that solutions approach the solution of the same problem without the highest derivative when t goes to infinity (that is the parabolic equation). This result has been generalized to the fractional derivative case in [7] and in [32].
For our problem (1), we would like to see how much influential ${}^C D_0^\beta y$ will be. In particular, how the range of values m ensuring non-existence would be affected. We reached the conclusion that here also it is the lower order derivative (i.e. β) which determines this range just like the parabolic part in the hyperbolic problem.
The rest of the paper is divided into three sections. In the next section, we present some definitions, notations, and lemmas which will be needed later in our proof. In Section 3, we present the test function and prove some properties for this function. Section 4 is devoted to the nonexistence result. In Section 5, we illustrate our findings by Numerical examples.

2 Preliminaries

In this section we present some definitions, lemmas, properties and notation which will be used in our result later.