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A numerical study for multiple solutions of a singular boundary value problem arising from laminar flow in a porous pipe with moving wall

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Abstract

This paper is concerned with multiple solutions of a singular nonlinear boundary value problem (BVP) on the interval $[0, 1]$, which arises in a study of the laminar flow in a porous pipe with an expanding or contracting wall. For the singular nonlinear BVP, the correct boundary conditions are derived to guarantee that its linearization has a unique smooth solution. Then a numerical technique is proposed to find all possible multiple solutions. For the suction driven pipe flow with the expanding wall (e.g. $\alpha = 2$), we find a new solution numerically and classify it as a type VI solution. The computed results agree well with what can be obtained by the bifurcation package AUTO. In addition, we also construct asymptotic solutions for a few cases of parameters, which agree well with numerical solutions. These serve as validations of our numerical results. Thus we believe that the numerical technique designed in the paper is reliable, and may be further applied to solve a variety of nonlinear equations that arise from other flow problems.

Keywords: singular boundary value problem; multiple solutions; singular perturbation method; expanding porous circular pipe

1. Introduction

The laminar flow in a porous circular pipe or channel with an expanding or contracting wall has received considerable attention in recent years due to their relevance to a number of biological and

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