

# Accepted Manuscript

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G. Barbatis, S. Filippas, A. Tertikas

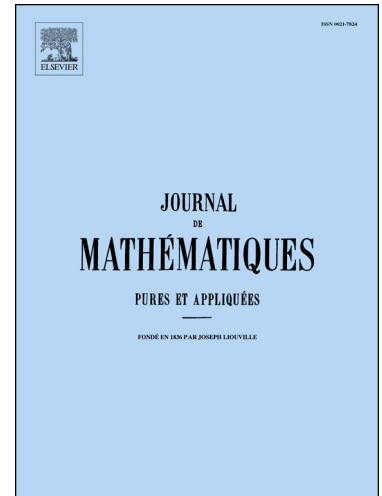
PII: S0021-7824(18)30074-6  
DOI: <https://doi.org/10.1016/j.matpur.2018.05.004>  
Reference: MATPUR 3003

To appear in: *Journal de Mathématiques Pures et Appliquées*

Received date: 14 February 2017

Please cite this article in press as: G. Barbatis et al., Sharp Hardy and Hardy–Sobolev inequalities with point singularities on the boundary, *J. Math. Pures Appl.* (2018), <https://doi.org/10.1016/j.matpur.2018.05.004>

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# Sharp Hardy and Hardy–Sobolev inequalities with point singularities on the boundary

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## Abstract

We study the Hardy inequality when the singularity is placed on the boundary of a bounded domain in  $\mathbb{R}^n$  that satisfies both an interior and exterior ball condition at the singularity. We obtain the sharp Hardy constant  $n^2/4$  in case the exterior ball is large enough and show the necessity of the large exterior ball condition. We improve Hardy inequality with the best constant by adding a sharp Sobolev term. We next produce criteria that lead to characterizing maximal potentials that improve Hardy inequality. Breaking the criteria one produces successive improvements with sharp constants. Our approach goes through in less regular domains, like cones. In the case of a cone, contrary to the smooth case, the Sobolev constant does depend on the opening of the cone.

## Résumé

Nous étudions l'inégalité de Hardy dans le cas où la singularité se trouve sur la frontière d'un domaine borné sur  $\mathbb{R}^n$  qui satisfait la fois une condition de boule intérieure et extérieure sur la singularité. Nous présentons la constante explicite de Hardy  $n^2/4$  obtenue dans le cas où la boule extérieure est suffisamment large et montrons la nécessité de la condition de la boule extérieure. Nous présentons une amélioration de l'inégalité de Hardy avec la meilleure constante en ajoutant un terme explicite de Sobolev. Par la suite, nous présentons certains critères capables de caractériser les potentiels maximaux qui améliorent l'inégalité de Hardy. En bafouant les critères nous produisons des améliorations successives avec des constantes explicites. Notre approche peut être appliquée dans des domaines moins réguliers, comme des cônes. Dans le cas d'une cône, contrairement au cas régulier, la constante de Sobolev dépend de l'ouverture de la cône.

2010 MSC : 35A23, 35J20, 35J75 (46E35, 26D10, 35J60)

*Keywords* : Hardy inequality, Hardy constant, boundary singularity, Sobolev inequality, maximal potential, best constant, conformality.

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<sup>1</sup>Partially supported by ELKE grant, University of Crete

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