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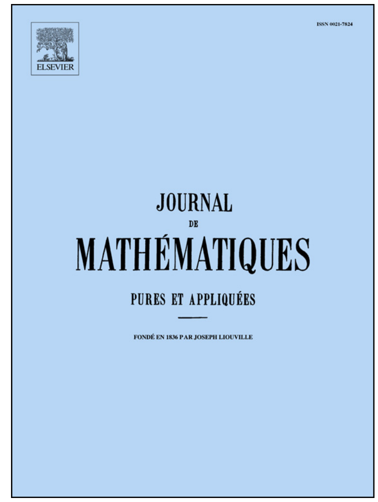

Direct and inverse elastic scattering from anisotropic media

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Direct and inverse elastic scattering from anisotropic media

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Abstract

Assume that a time-harmonic elastic wave is incident onto a penetrable anisotropic elastic body embedded in a homogeneous isotropic background medium. The scattering problem is reduced to a truncated domain. Uniqueness and existence of weak solutions are proved by applying the Fredholm alternative and using properties of the Dirichlet-to-Neumann map in both two and three dimensions. The Fréchet derivative of the near-field solution operator with respect to the boundary of the scatterer is derived. As an application, a descent algorithm is designed for recovering the interface from the near-field data of one or several incident directions and frequencies. Numerical examples in 2D are presented to show the validity and accuracy of the algorithm.

Keywords: Elasticity, Lamé system, variational approach, Fréchet derivative, Dirichlet-to-Neumann map, inverse scattering.

2010 MSC: 35J20, 65K10, 74J20

1. Introduction

Time-harmonic elastic scattering problems arise from many mechanic systems and engineering structures, in which the linear elasticity theory ([1]) provides an essential tool for analysis and design. For an infinite background medium, the boundary value problem for the Lamé system can be reduced to an equivalent system on a bounded domain. For instance, the finite element method for the scattering problems usually requires a strongly elliptic variational formulation with a nonlocal boundary condition (see e.g.,[2, 3]). To truncate the unbounded domain, one needs to derive the so-called Dirichlet-to-Neumann map (or non-reflecting boundary condition, transparent boundary operator) on an artificial boundary as a replacement of the Kupradze radiation condition at infinity. In the literature, the DtN map in elastodynamics have been used by some physicists and engineers for simulation ([4, 5, 6, 7, 8, 9, 10]). However, properties of the

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