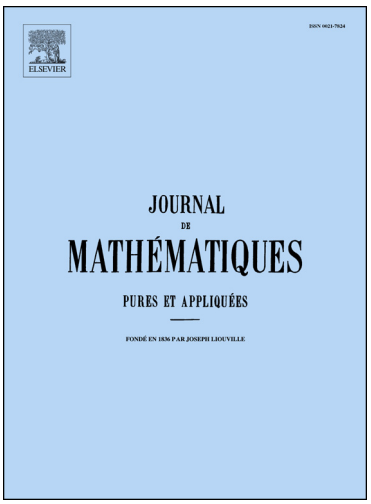

L^p-maximal hypoelliptic regularity of nonlocal kinetic Fokker-Planck operators

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L^p-maximal hypoelliptic regularity of nonlocal kinetic Fokker-Planck operators ☆

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Abstract

For $p \in (1, \infty)$, let $u(t, x, \mathrm{v})$ and $f(t, x, \mathrm{v})$ be in $L^p(\mathbb{R}\times\mathbb{R}^d\times\mathbb{R}^d)$ and satisfy the following nonlocal kinetic Fokker-Plank equation on $\mathbb{R}^{1+2d}$ in the weak sense:

$$\partial_t u + \mathrm{v}\cdot\nabla_x u = \Delta_{\mathrm{v}}^{\alpha/2} u + f,$$

where $\alpha \in (0, 2)$ and $\Delta_{\mathrm{v}}^{\alpha/2}$ is the usual fractional Laplacian applied to v-variable. We show that there is a constant $C = C(p, \alpha, d) > 0$ such that for any $f(t, x, \mathrm{v}) \in L^p(\mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d) = L^p(\mathbb{R}^{1+2d})$,

$$\|\Delta_x^{\alpha/(2(1+\alpha))} u\|_p + \|\Delta_{\mathrm{v}}^{\alpha/2} u\|_p \leqslant C\|f\|_p,$$

where $\|\cdot\|_p$ is the usual L^p-norm in $L^p(\mathbb{R}^{1+2d}; \mathrm{d}z)$. In fact, in this paper the above inequality is established for a large class of time-dependent non-local kinetic Fokker-Plank equations on $\mathbb{R}^{1+2d}$, with $U_t\mathrm{v}$ and $\mathscr{L}_{\sigma_t}^{\nu_t}$ in place of $\mathrm{v}\cdot\nabla_x$ and $\Delta_{\mathrm{v}}^{\alpha/2}$. See Theorem 3.3 for details.

Résumé. Pour $p \in (1, \infty)$, let $u(t, x, \mathrm{v})$ et $f(t, x, \mathrm{v})$ être dans $L^p(\mathbb{R}\times\mathbb{R}^d\times\mathbb{R}^d)$ et satisfaire l'équation de la Fokker-Plank kinétique non locale sur $\mathbb{R}^{1+2d}$ au sens faible:

$$\partial_t u + \mathrm{v}\cdot\nabla_x u = \Delta_{\mathrm{v}}^{\alpha/2} u + f,$$

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