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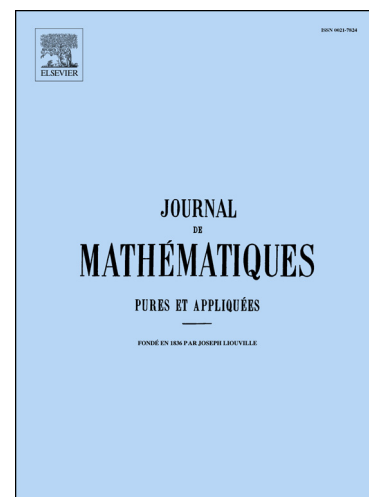
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# $L^p$ -maximal hypoelliptic regularity of nonlocal kinetic Fokker-Planck operators <sup>☆</sup>

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## Abstract

For  $p \in (1, \infty)$ , let  $u(t, x, v)$  and  $f(t, x, v)$  be in  $L^p(\mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d)$  and satisfy the following nonlocal kinetic Fokker-Planck equation on  $\mathbb{R}^{1+2d}$  in the weak sense:

$$\partial_t u + v \cdot \nabla_x u = \Delta_v^{\alpha/2} u + f,$$

where  $\alpha \in (0, 2)$  and  $\Delta_v^{\alpha/2}$  is the usual fractional Laplacian applied to  $v$ -variable. We show that there is a constant  $C = C(p, \alpha, d) > 0$  such that for any  $f(t, x, v) \in L^p(\mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d) = L^p(\mathbb{R}^{1+2d})$ ,

$$\|\Delta_x^{\alpha/(2(1+\alpha))} u\|_p + \|\Delta_v^{\alpha/2} u\|_p \leq C \|f\|_p,$$

where  $\|\cdot\|_p$  is the usual  $L^p$ -norm in  $L^p(\mathbb{R}^{1+2d}; dz)$ . In fact, in this paper the above inequality is established for a large class of time-dependent non-local kinetic Fokker-Planck equations on  $\mathbb{R}^{1+2d}$ , with  $U_t v$  and  $\mathcal{L}_{\sigma_t}^{v_t}$  in place of  $v \cdot \nabla_x$  and  $\Delta_v^{\alpha/2}$ . See Theorem 3.3 for details.

**Résumé.** Pour  $p \in (1, \infty)$ , let  $u(t, x, v)$  et  $f(t, x, v)$  être dans  $L^p(\mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d)$  et satisfaire l'équation de la Fokker-Planck cinétique non locale sur  $\mathbb{R}^{1+2d}$  au sens faible:

$$\partial_t u + v \cdot \nabla_x u = \Delta_v^{\alpha/2} u + f,$$

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