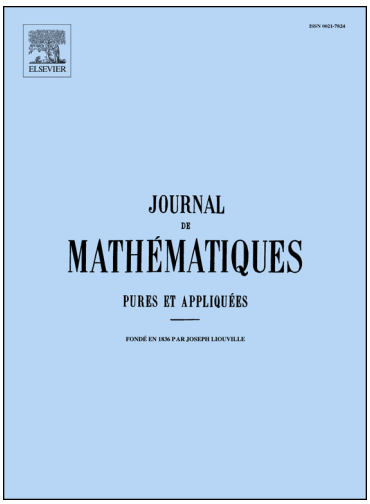


# Gradient estimates for elliptic equations with measurable nonlinearities

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# Gradient estimates for elliptic equations with measurable nonlinearities

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**Abstract**

We obtain Calderón-Zygmund type estimate for nonlinear elliptic equations of $p$-Laplacian type, under the condition that the associated nonlinearity is allowed to be merely measurable in one spatial variable, but has locally small mean oscillation in the remaining spatial variables. This is the minimal regularity requirement on the associated nonlinearity for Calderón-Zygmund type estimate, in the sense that if the associated nonlinearity is allowed to be merely measurable with respect to two independent spatial variables then Calderón-Zygmund type estimate fails in general.

**Résumé**

Nous obtenons une estimation du type Calderón-Zygmund pour des équations elliptiques $p$-Laplaciennes avec la condition que la non-linéarité associée est permise d'être seulement mesurable en dimension d'un espace, mais avoir une petite oscillation moyenne localement dans les dimensions supérieures. Celle-ci est la régularité minimale sur la non-linéarité pour des estimations du type Calderón-Zygmund dans le sens où si la non-linéarité associée est permise d'être seulement mesurable par rapport aux deux spatiales variables, alors l'estimation du type Calderón-Zygmund échoue en général.

*Keywords:* Calderón-Zygmund type estimates, Nonlinear elliptic equations, Measurable nonlinearities
*2010 MSC:* Primary 35J60, 35B65 Secondary 35R05

## 1. Introduction

We study nonlinear elliptic equations of $p$-Laplacian type

$$\text{div } a(Du, x) = \text{div } (|F|^{p-2}F) \quad \text{in } \Omega \qquad p \in (1, \infty),$$

to find a minimal regularity requirement on the nonlinearity $a$, under which we establish the classical Calderón-Zygumnd type theory

$$F \in L^q_{loc}(\Omega) \quad \Longrightarrow \quad Du \in L^q_{loc}(\Omega) \qquad q \in [p, \infty). \tag{1}$$

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