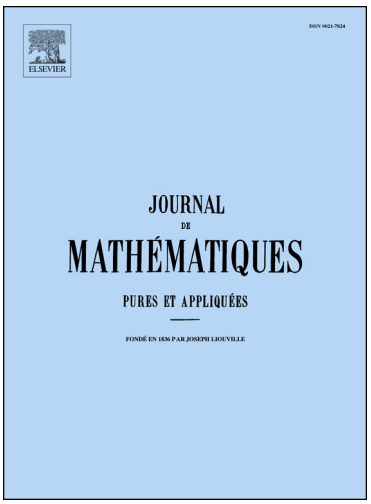

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Large time monotonicity of solutions of reaction-diffusion equations in $\mathbb{R}^N$

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Abstract

In this paper, we consider nonnegative solutions of spatially heterogeneous reaction-diffusion equations in the whole space. Under some assumptions on the initial conditions, including in particular the case of compactly supported initial conditions, we show that, above any arbitrary positive value, the solution is increasing in time at large times. Furthermore, in the one-dimensional case, we prove that, if the equation is homogeneous outside a bounded interval and the reaction is linear around the zero state, then the solution is time-increasing in the whole line at large times. The question of the monotonicity in time is motivated by a medical imagery issue.

Dans cet article nous étudions les solutions positives d'équations de réaction diffusion dans l'espace entier. Sous certaines conditions sur la donnée initiale, nous démontrons que, au dessus d'une certaine valeur arbitrairement petite, la solution est croissante en temps pour des temps assez grands.

Keywords: Reaction diffusion equation, monotonicity properties, qualitative analysis
2000 MSC: 35K57

1. Introduction and main results

In this paper, we consider the Cauchy problem for the following reaction-diffusion equation set in the whole space $\mathbb{R}^N$

$$\begin{cases} u_t &= \mathrm{div}(A(x)\nabla u) + f(x,u), \quad t>0,\ x\in\mathbb{R}^N, \\ u(0,x) &= u_0(x). \end{cases} \tag{1.1}$$

Here u_t stands for $u_t(t,x) = \frac{\partial u}{\partial t}(t,x)$ and the divergence and the gradient act on the spatial variables x. We are interested in the monotonicity in time for large times, when

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