



Contents lists available at ScienceDirect

Journal de Mathématiques Pures et Appliquées

www.elsevier.com/locate/matpur



Perfect transmission invisibility for waveguides with sound hard walls

Anne-Sophie Bonnet-Ben Dhia^a, Lucas Chesnel^{b,*}, Sergei A. Nazarov^{c,d,e}

^a Laboratoire Poems, CNRS/ENSTA/INRIA, Ensta ParisTech, Université Paris-Saclay, 828, boulevard des Maréchaux, 91762 Palaiseau, France

^b INRIA/Centre de mathématiques appliquées, École polytechnique, Université Paris-Saclay, route de Saclay, 91128 Palaiseau, France

^c St. Petersburg State University, Universitetskaya Naberezhnaya, 7-9, 199034 St. Petersburg, Russia

^d Peter the Great St. Petersburg Polytechnic University, Polytekhnicheskaya ul., 29, 195251 St. Petersburg, Russia

^e Institute of Problems of Mechanical Engineering, Bolshoy Prospekt, 61, 199178, V.O., St. Petersburg, Russia

ARTICLE INFO

Article history:

Received 25 September 2016

Available online xxxx

MSC:

35R30

65N21

65N25

35J05

78A46

78A40

Keywords:

Invisibility

Acoustic waveguide

Asymptotic analysis

Scattering matrix

ABSTRACT

We are interested in a time harmonic acoustic problem in a waveguide with locally perturbed sound hard walls. We consider a setting where an observer generates incident plane waves at $-\infty$ and probes the resulting scattered field at $-\infty$ and $+\infty$. Practically, this is equivalent to measure the reflection and transmission coefficients respectively denoted R and T . In [9], a technique has been proposed to construct waveguides with smooth walls such that $R = 0$ and $|T| = 1$ (non-reflection). However the approach fails to ensure $T = 1$ (perfect transmission without phase shift). In this work, first we establish a result explaining this observation. More precisely, we prove that for wavenumbers smaller than a given bound k_* depending on the geometry, we cannot have $T = 1$ so that the observer can detect the presence of the defect if he/she is able to measure the phase at $+\infty$. In particular, if the perturbation is smooth and small (in amplitude and in width), k_* is very close to the threshold wavenumber. Then, in a second step, we change the point of view and, for a given wavenumber, working with singular perturbations of the domain, we show how to obtain $T = 1$. In this case, the scattered field is exponentially decaying both at $-\infty$ and $+\infty$. We implement numerically the method to provide examples of such undetectable defects.

© 2017 Elsevier Masson SAS. All rights reserved.

R É S U M É

Nous nous intéressons à un problème d'acoustique en régime harmonique en temps dans un guide d'ondes dont les parois dures sont localement perturbées. Nous considérons une configuration dans laquelle un observateur génère une onde plane incidente en $-\infty$ et mesure le champ diffracté résultant en $-\infty$ et en $+\infty$. En pratique, cela revient à mesurer les coefficients de réflexion et transmission, respectivement notés R et T . Dans l'article [9], les auteurs proposent une technique

* Corresponding author.

E-mail addresses: Anne-Sophie.Bonnet-Bendhia@ensta-paristech.fr (A.S. Bonnet-Ben Dhia), lucas.chesnel@inria.fr (L. Chesnel), srgnazarov@yahoo.co.uk (S.A. Nazarov).

permettant de construire des guides d'ondes à parois molles tels que $R = 0$ et $|T| = 1$ (non-réflexion). Cependant, l'approche ne permet pas d'imposer $T = 1$ (transmission parfaite sans déphasage). Dans ce travail, nous établissons d'abord un résultat permettant d'expliquer cette observation. Plus précisément, nous prouvons que, pour des nombres d'onde plus petits qu'une borne donnée k_* dépendant de la géométrie, on ne peut avoir $T = 1$, de sorte qu'un observateur peut détecter la présence du défaut s'il est capable de mesurer la phase en $+\infty$. En particulier, si la perturbation est régulière et petite (en amplitude ainsi qu'en largeur), la borne k_* est très proche du nombre d'onde seuil. Dans un second temps, nous changeons de point de vue et, pour un nombre d'onde donné, en travaillant avec des perturbations singulières du domaine, nous montrons comment obtenir $T = 1$. Dans une telle situation, le champ diffracté est exponentiellement décroissant en amplitude à la fois en $-\infty$ et en $+\infty$. Nous mettons en œuvre numériquement la méthode pour fournir des exemples de tels défauts indétectables.

© 2017 Elsevier Masson SAS. All rights reserved.

1. Introduction

Invisibility is a very dynamic field of research in inverse scattering theory. In this article, we are interested in a time harmonic acoustic problem in a waveguide with a bounded transverse section. There is an important literature concerning techniques of imaging for waveguides (see e.g. [52,51,13,2,38,12,11]). Here, we consider a situation where an observer wants to detect the presence of defects in some reference waveguide from far-field data. We assume that the observer is located far from the defect. Practically, the observer generates waves, say from $-\infty$, and measures the amplitude of the resulting scattered field at $\pm\infty$. It is known that, at a given frequency, at $\pm\infty$, the scattered field decomposes as the sum of a finite number of propagative waves plus some exponentially decaying remainder. In this work, we will assume that the frequency is sufficiently small so that only one wave (the piston mode for the problem considered here with sound hard walls) can propagate in the waveguide. In this case, one usually introduces two complex coefficients, namely the *reflection* and *transmission* coefficients, denoted R and T , such that R (resp. $T - 1$) corresponds to the amplitude of the scattered field at $-\infty$ (resp. $+\infty$) (see (11)). According to the energy conservation, we have

$$|R|^2 + |T|^2 = 1. \quad (1)$$

We shall say that the defects in the reference waveguide are perfectly invisible at a given frequency if there holds $R = 0$ and $T = 1$. In such a situation, the scattered field is exponentially decaying at $\pm\infty$ and the observer cannot detect the presence of the defect from noisy measurements.

In this context, examples of quasi invisible obstacles ($|R|$ small or $|T - 1|$ small), obtained via numerical simulations, exist in literature. We refer the reader to [20] for a water waves problem and to [1,18,48,49,22] for strategies based on the use of new “zero-index” and “epsilon near zero” metamaterials in electromagnetism (see [21] for an application to acoustic). Let us mention also that the problem of the existence of quasi invisible obstacles for frequencies close to the threshold frequency has been addressed in the analysis of the so-called Weinstein anomalies [54] (see e.g. [46,35]).

Recently in [9,8], (see also [10,7,15,16] for applications to other problems) a technique has been proposed to prove the existence of waveguides different from the straight geometry $\mathbb{R} \times \omega$, where ω is the cross section, with sound hard walls such that $R = 0$ (rigorously). If an observer located at $-\infty$ generates a propagative wave in such a waveguide, then the scattered field is exponentially decaying at $-\infty$. Therefore, the perturbation of the walls is invisible for backscattering measurements. These waveguides are said to be *non-reflecting*. Now, imagine that the observer located at $-\infty$ can also measure the transmission coefficient T . As a consequence of formula (1), when $R = 0$, we have $|T| = 1$. In this case, if the observer measures only the amplitude of waves (the modulus of T), again the perturbation is invisible. But if he/she can measure the phase of waves

Download English Version:

<https://daneshyari.com/en/article/8902420>

Download Persian Version:

<https://daneshyari.com/article/8902420>

[Daneshyari.com](https://daneshyari.com)