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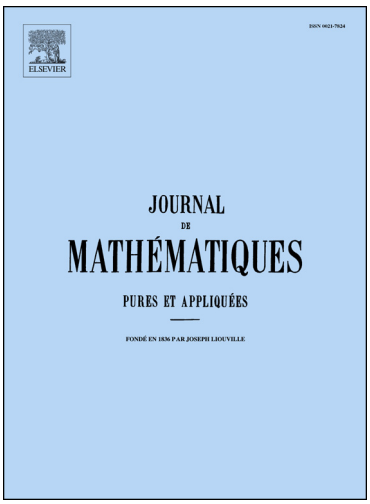

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Approximate Correctors and Convergence Rates in Almost-Periodic Homogenization

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Abstract

We carry out a comprehensive study of quantitative homogenization of second-order elliptic systems with bounded measurable coefficients that are almost-periodic in the sense of H. Weyl. We obtain uniform local L^2 estimates for approximate correctors in terms of a function that quantifies the almost-periodicity of the coefficient matrix. We give a condition that implies the existence of (true) correctors. These estimates as well as similar estimates for the dual approximate correctors yield optimal or near optimal convergence rates in H^1 and L^2. The L^2-based Hölder and Lipschitz estimates at large scale are also established.

Keywords:
Homogenization, Almost Periodic, Approximate Correctors, Convergence Rates.
2010 MSC: 35B27, 74Q20.

1. Introduction

In this paper we shall be interested in quantitative homogenization of a family of second-order elliptic operators with rapidly oscillating almost-periodic coefficients,

$$\mathcal{L}_\varepsilon = -\text{div}(A(x/\varepsilon)\nabla) = -\frac{\partial}{\partial x_i}\left\{a_{ij}^{\alpha\beta}\left(\frac{x}{\varepsilon}\right)\frac{\partial}{\partial x_j}\right\}, \qquad \varepsilon > 0 \tag{1.1}$$

(the summation convention is used throughout). We assume that the coefficient matrix $A(y) = (a_{ij}^{\alpha\beta}(y))$ with $1 \le i, j \le d$ and $1 \le \alpha, \beta \le m$ is real, bounded measurable, and satisfies the ellipticity condition,

$$\mu|\xi|^2 \le a_{ij}^{\alpha\beta}(y)\xi_i^\alpha \xi_j^\beta \le \mu^{-1}|\xi|^2 \quad \text{for a.e. } y \in \mathbb{R}^d \text{ and } \xi = (\xi_i^\alpha) \in \mathbb{R}^{m\times d}, \tag{1.2}$$

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