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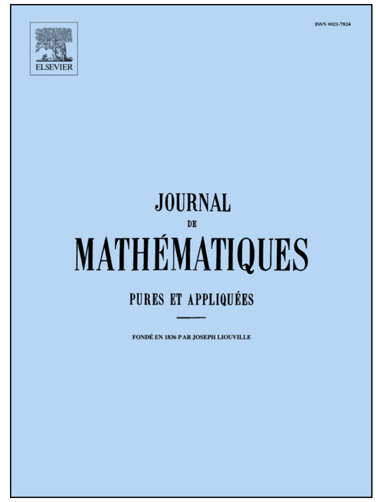

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ENERGY TRANSFER BETWEEN MODES IN A NONLINEAR BEAM EQUATION

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Abstract. We consider the nonlinear nonlocal beam evolution equation introduced by Woinowsky-Krieger [38]. We study the existence and behavior of periodic solutions: these are called nonlinear modes. Some solutions only have two active modes and we investigate whether there is an energy transfer between them. The answer depends on the geometry of the energy function which, in turn, depends on the amount of compression compared to the spatial frequencies of the involved modes. Our results are complemented with numerical experiments; overall, they give a complete picture of the instabilities that may occur in the beam. We expect these results to hold also in more complicated dynamical systems.

Résumé: On considère l'équation d'évolution de la poutre nonlinéaire et nonlocale introduite par Woinowsky-Krieger [38]. On étudie l'existence et le comportement des solutions périodiques: on les appelle modes nonlinéaires. Certaines solutions ont seulement deux modes actifs et nous étudions le possible transfer d'énergie entre eux. La réponse dépend de la géométrie de la fonctionnelle d'énergie qui, à son tour, dépend de la quantité de compression et des fréquences spatiales des modes actifs. Nos résultats sont complétés par des experiments numériques; ils donnent une description d'ensemble assez complète des instabilités qui peuvent apparaître dans la poutre. On s'attend à ce que ces résultats soient valables aussi pour des systèmes dynamiques plus compliqés.

Keywords: nonlinear beam equation, energy transfer between modes, stability, compression.
AMS Subject Classification (2010): 35G31, 34D20, 35A15, 74B20, 74K10.

1. Introduction

In 1950, Woinowsky-Krieger [38] modified the classical beam models by Bernoulli and Euler assuming a nonlinear dependence of the axial strain on the deformation gradient, by taking into account the stretching of the beam due to its elongation. Let us mention that, independently, Burgreen [9] derived the very same nonlinear beam equation which reads

$$M\,u_{tt}+EI\,u_{xxxx}+\Big[P-\eta\,\|u_x\|^2_{L^2(0,\ell)}\Big]u_{xx}=f\qquad x\in(0,\ell)\,,\ t>0\,,$$

where u denotes the vertical displacement of the beam whose length is ℓ. The constant $\eta>0$ depends on the elasticity of the material composing the beam and the term $\eta\|u_x\|^2_{L^2(0,\ell)}$ measures the geometric nonlinearity of the beam due to its stretching. The constant P is the axial force acting at the endpoints of the beam: a positive P means that the beam is compressed while a negative P means that the beam is stretched. We are mainly interested in compressed beams ($P>0$) although some of our results also apply to free ($P=0$) and stretched ($P<0$) beams. Finally, $M>0$ denotes the mass per unit length, $EI>0$ is the flexural rigidity of the beam, whereas $f=f(x,t)$ is an external load.

We assume that the beam is hinged at its endpoints and this results in the so-called Navier boundary conditions. For simplicity, we consider a beam lying on the segment $x\in(0,\pi)$, we normalize the constants, we take null force, and we reduce to

$$\begin{cases} u_{tt}+u_{xxxx}+\Big[P-\frac{2}{\pi}\,\|u_x\|^2_{L^2(0,\pi)}\Big]u_{xx}=0 & x\in(0,\pi)\,,\ t>0\,,\\ u(0,t)=u(\pi,t)=u_{xx}(0,t)=u_{xx}(\pi,t)=0 & t>0\,. \end{cases}\tag{1}$$

A description of (1) with $f\neq 0$ would require a huge effort and falls beyond the scopes of this paper. We expect this kind of analysis to require the exploitation of previous results for related forced ODE's, see for instance [10, 17]. The existence and uniqueness of global solutions of the initial value problem